Area Under A Bridge Teacher Notes

Topic Area: Curve fitting to image

NCTM Standards:

- Create and use representations to organize, record, and communicate mathematical ideas.
- Use Mathematical models to represent and understand quantitative relationships.

Objective

Given a photo file, students will be able to fit an equation on to it. Using their knowledge of polynomial functions and integral calculus, students will find the area under the curve using different methods, including Riemann sums, trapezoidal and midpoint rule.

Getting Started

Have students work in pairs to help determine what points should be plotted and to facilitate discussion of the questions.

Prior to using this activity:

- Students should have a basic understanding of what regression is and what it does.
- Students should understand the meaning of the Riemann sums and the trapezoidal rule.
- Students should be able to find the area of a rectangle and trapezoid.
- Students should be able calculate an output value given an input value and calculate the input value given an output value.

Ways students can provide evidence of learning:

- Students will be able to use their knowledge of geometry to find areas.
- Fill in the table to find areas.
- Find y values for midpoint values of x.

Common mistakes to be on the lookout for:

- Students may be careless in the placement of points.
- Students often have difficulty with the midpoint method.
- Students have difficulty with finding and using the correct y value or use the wrong value.

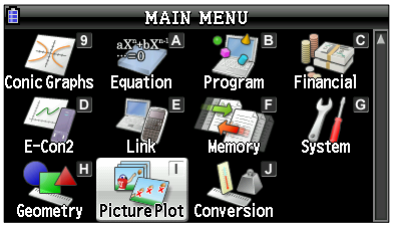
Definitions

- Trapezoidal rule
- Riemann sum
- Integral
- Regression

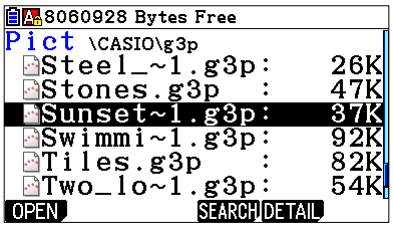
Area Under A Bridge "How To"

The following will walk you through the keystrokes and menus required to successfully complete the Area under a bridge activity.

To open a background image in Picture Plot:

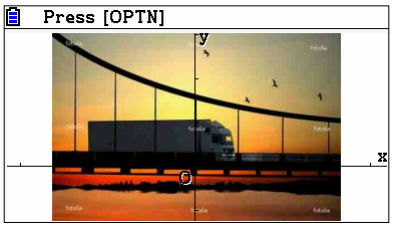


1. From the Main Menu, highlight the Picture Plot icon and press EXE or press (.



2. Press F1 (OPEN) to open the CASIO folder.

3. The g3p folder contains 47 background images. Press ▼ F1 (OPEN) to open the folder. Scroll down the list of images and highlight the desired image. You will be using the "Sunset~1" image in this activity. Press F1 (OPEN).

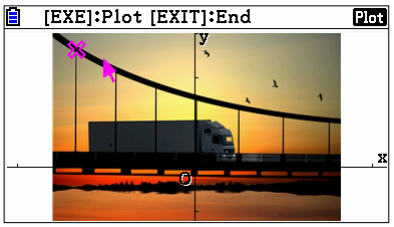


To plot points on the image and create a list of points:

1. The status bar at the top of the screen prompts what buttons you have to choose from. For this picture, you will need to press OPTN .

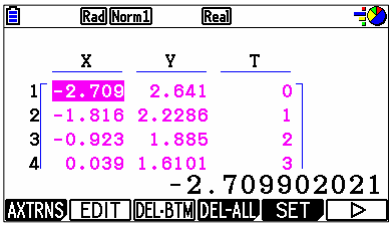
2. To plot points on the picture, press [F2] (Plot).
A pink arrow will appear; use ◀ ▶ ▲ ▼ to move the arrow to where you would like for it to plot a point. (Any of the number keys can also be used to jump to different areas on the screen).
Press [EXE] to plot the point on the picture.

3. Continue moving the arrow and pressing [EXE] until you have all the points you want (one on the end of each suspension cable).
To stop plotting, press [EXIT].



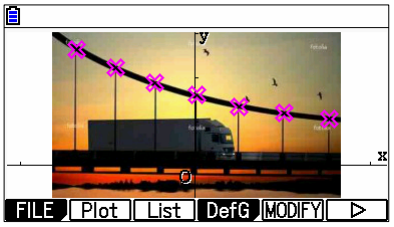
To view the list of data points:

1. Press [OPTN] [F3] to view the list of points plotted.
Press [EXIT] to go back to the picture and points.

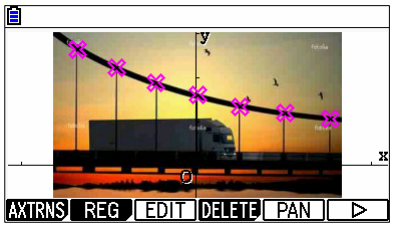


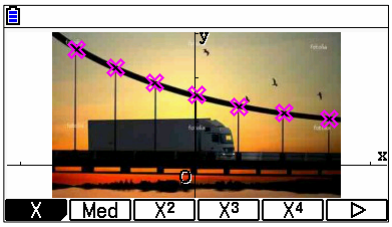
To create a best fit line or curve of best fit:

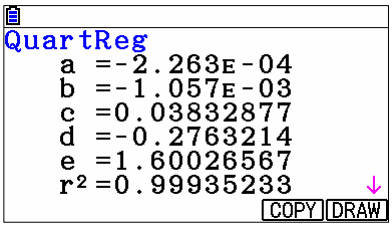
1. Press [F6] (▷) and [F2] (REG).



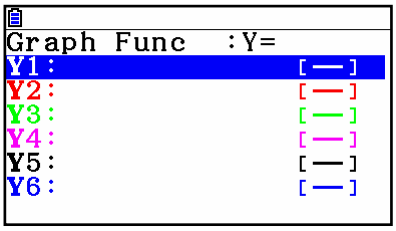
2. Choose the appropriate regression model.
In this case, it will be X^4 so press [F5].



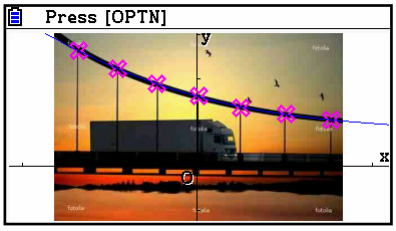




3. Press F5 (Copy) and EXE to copy the equation to the Graph menu.

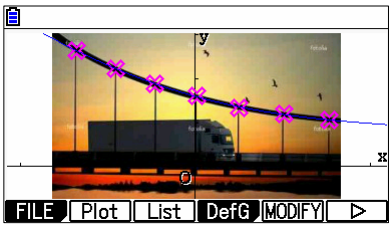


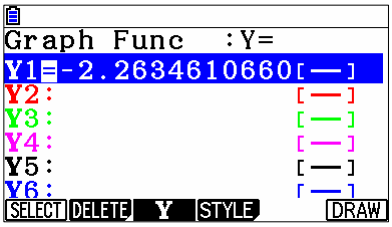
4. Press F6 (DRAW) to see the regression curve and the points.



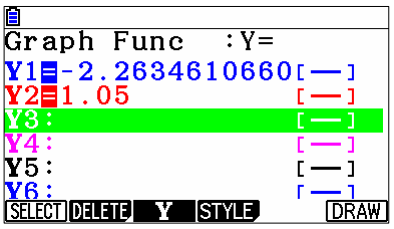
To find the intersection with the top of the truck:

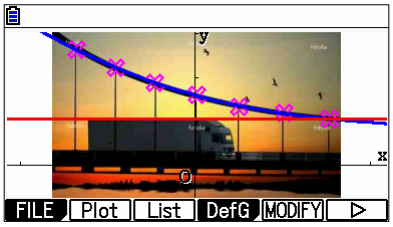
1. To enter another equation, press OPTN F4 (DefG).



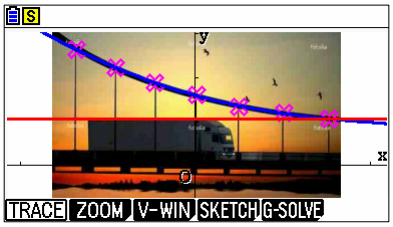


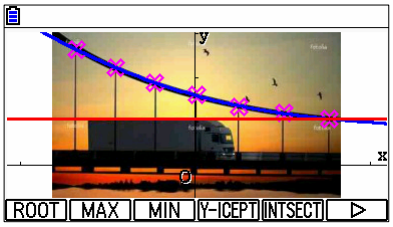
2. Press ▼ and enter 1 • 0 5 EXE F6 (DRAW).

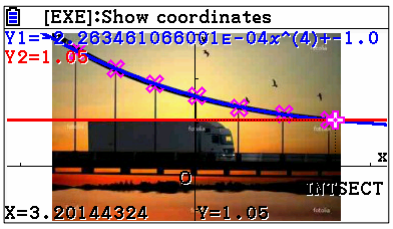




3. To find the intersection, press SHIFT F5 **(G-SOLVE)** F5 (INTSECT). Select curve and line, pressing EXE after each.

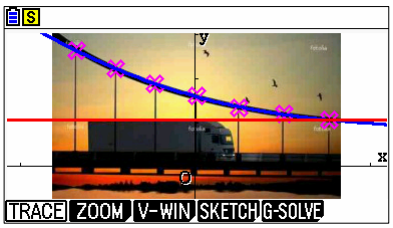


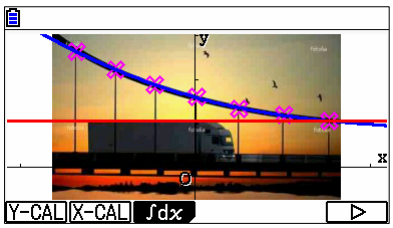


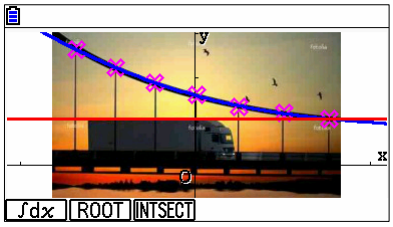


To find the integral:

1. Press SHIFT F5 (G-Solv) F6 (▷) F3 (∫dx) F1.

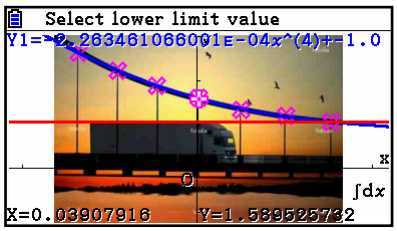


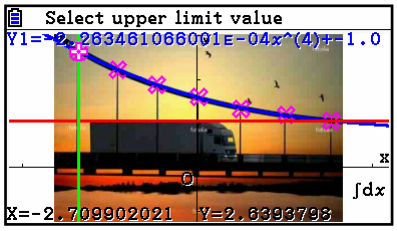


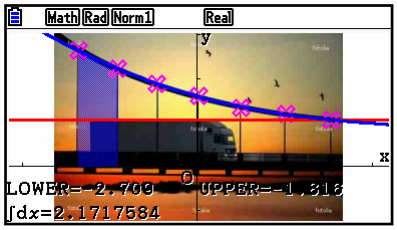


2. If there is more than one equation graphed, the calculator will ask you to specify which graph to find the integral of. Use the ▲ ▼ keys and press EXE to choose the desired graph.

3. The calculator will prompt you to select the lower limit value and the upper limit value. Use the ◀ ▶ keys or input a value and press EXE.







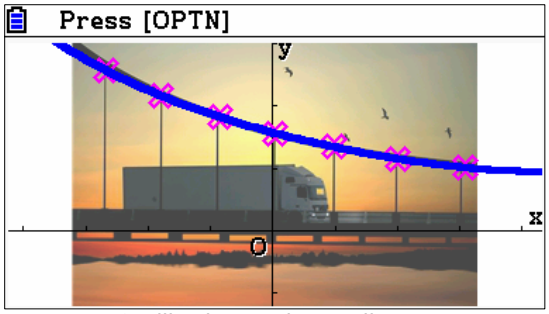
Area Under A Bridge Activity

Communities are trying to improve noise pollution on highways by installing noise reducing barriers. These barriers block sounds form traffic, but the barriers are expensive and must be custom made. In the picture below, you will need to find the area under the bridge cables to estimate cost and reduce waste.

Questions

1. List the coordinates of the support cables

X coordinate	Y coordinate
1)	1)
2)	2)
3)	3)
4)	4)
5)	5)
6)	6)
7)	7)

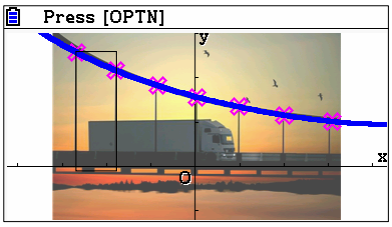


2. What is the equation of the best fit line to the curved bridge cable?

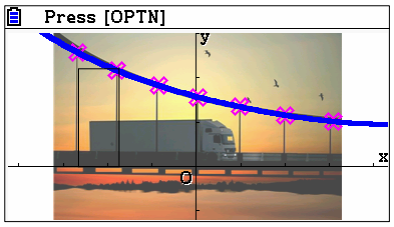
_____

3. Using the coordinates find the right sided, left sided, midpoint and trapezoidal estimate of the area under the curve. (Remember- the width is the difference in x coordinates and the length is the height or the y coordinates.)

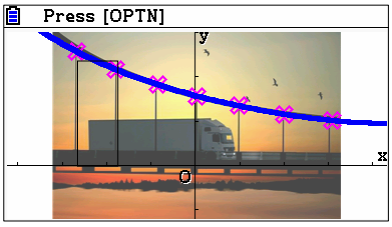
Width	Length	Right Sided Area	Left sided Area	Midpoint Area	Trapezoidal Area
		Sum:	**Sum:**	**Sum:**	**Sum:**



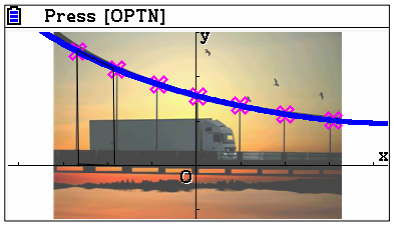
Right sided



Left sided



Midpoint



Trapezoidal

4. Using the integral feature, find the area under the curve

5. Which estimate is the best? Explain.

_____

_____

Extension

1. What is the maximum height a truck can be to make it under the cable?

_____

2. Will the truck make it under the cable? If not at what point will it make contact with the cable?

_____

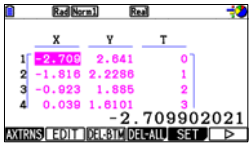
_____

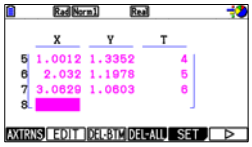
3. Find an equation that connects all four of the birds in the picture.

_____

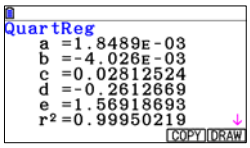
Solutions

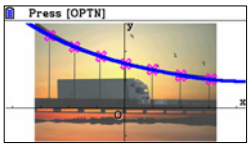
1. Answers may vary, depending on the plotted points.





2. Answers may vary, depending on the plotted points.





3. Width = x-values
Length = y-values
Right Sided Area = $(x_2 - x_1) \bullet y_1$
Left Sided Area = $(x_2 - x_1) \bullet y_2$
Midpoint Area = $(x_2 - x_1) \bullet \mathbf{Y1}\left(\frac{x_1 + x_2}{2}\right)$
Trapezoidal Area = $\frac{y_1 + y_2}{2} \bullet (x_2 - x_1)$

Width	Length	Right Sided Area	Left sided Area	Midpoint Area	Trapezoidal Area
-2.71	2.64	2.35	1.98	2.16	2.17
-1.82	2.23	2.01	1.70	1.85	1.85
-0.92	1.89	1.66	1.42	1.53	1.54
-0.04	1.61	1.67	1.39	1.54	1.53
1.00	1.34	1.38	1.24	1.30	1.31
2.03	1.20	1.24	1.09	1.15	1.16
3.06	1.06	**Sum:** 10.31	**Sum:** 8.82	**Sum:** 9.53	**Sum:** 9.56

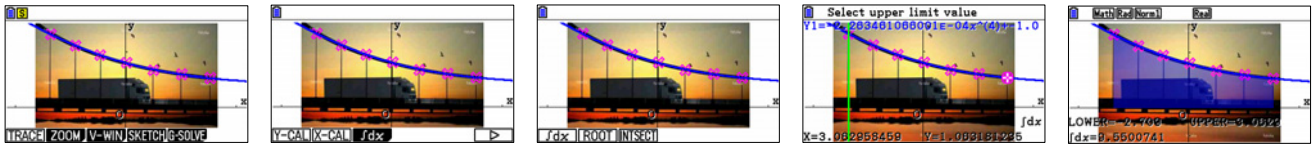
Right Sided Area = (-1.82 – -2.71)•2.64 = 2.3496
Left Sided Area = (-1.82 – -2.71)•2.23 = 1.9847

Midpoint Area = (-1.8 – -2.67)•**Y1**$(\frac{-2.67+-1.80}{2})$ = (0.89)•2.4291 = 2.1619

Y1(-2.265) = 2.4291

Trapezoidal Area = $\frac{2.64+2.23}{2}$•(-1.82 – -2.71) = 2.1672

4. Area under the curve is 9.55.



5. The Trapezoidal rule is the best estimator for area under the curve. It was off by 0.01 units^2. The Midpoint rule was a close second, off by 0.02 units^2.

Extension Solutions

1. The maximum height for a truck is the minimum value of the function at y = 1.08.

2. Answers will vary, as long as the truck's estimated height is below the minimum of 1.08 it will. A good estimate is 1.10 for the height. In that case, the truck will hit the wire at x = 2.51.

3. One possible solution: