

## Area of Similar Figures

## Teacher Notes

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**Topic Area:** Ratios of Similar Figures

**NCTM Standards:**

- Use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume.
- Use Cartesian coordinates and other coordinate systems, such as navigational, polar, or spherical systems, to analyze geometric situations.
- Investigate conjectures and solve problems involving two- and three-dimensional objects represented with Cartesian coordinates.

**Objectives**

Given a photo file, students will be able to find the coordinates for the endpoints of a radius for several circular objects. Using their knowledge of coordinate geometry and proportions, students will compare the ratio of the circumference and area to the radius for several circles.

**Getting Started**

Have students work in pairs to determine the coordinates of the endpoints of the radius for several circles and use these coordinates to calculate the length of the radius. Using the radius, students will calculate the circumference and area of the circles and determine the relationship between the ratio of the radii and the measures of the circumference and area.

**Prior to using this activity:**

- Students should be able to calculate the distance between two points.
- Students should know the formulas for finding the circumference and area of a circle.
- Students should be able to write a ratio and proportion.

**Ways students can provide evidence of learning:**

- Students will be able to calculate the length of the radius using the coordinates of the endpoints.
- Students will be able to compare the ratios between measures and write a conjecture.

**Common mistakes to be on the lookout for:**

- Students may enter values into the formulas incorrectly.
- Students may be careless in choosing the endpoints of the radius.
- Students may write the proportions incorrectly in order to comparing the relationships.

**Definitions:**

- Radius
- Circumference
- Area

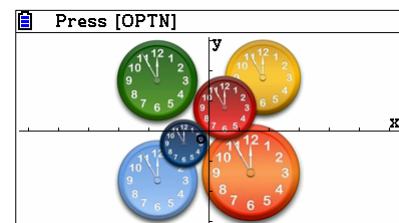
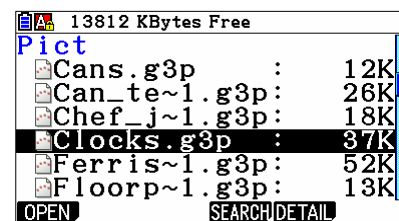
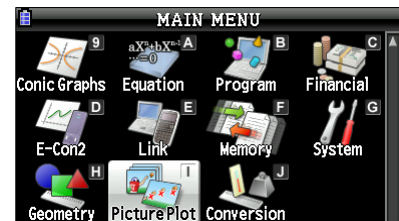
## Area of Similar Figures

## “How To”

The following will walk you through the keystrokes and menus required to successfully complete the area of similar figures activity.

### To open a background image in Picture Plot:

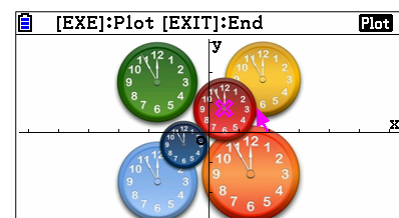
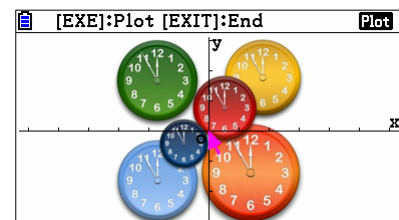
1. From the Main Menu, highlight the Picture Plot icon and press **EXE** or press **□**.
2. Press **F1** (OPEN) to open the CASIO folder.
3. The g3p folder contains 47 background images. Press **▼** **F1** (OPEN) to open the folder. Scroll down the list of images and highlight the desired image. You will be using the “Clocks” image in this activity. Press **F1** (OPEN).







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### To find the coordinates of the endpoints for the radius:

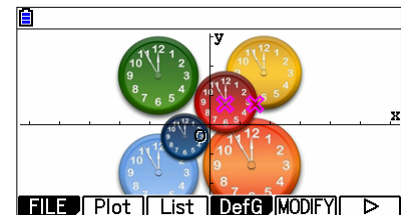
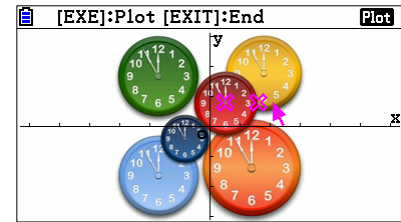
1. The status bar at the top of the screen prompts what buttons you have to choose from. For this image, you need to press **OPTN**.
2. To plot points on the picture, press **F2** (Plot). A pink arrow will appear. Use **◀** **▶** **▲** **▼** to move the arrow to the center of the red clock. (Any of the number keys can also be used to jump to different areas on the screen). Press **EXE** to plot the point on the image.



- Use     to move the arrow to the edge of the red clock and press **EXE** to plot the point on the image. You now have the endpoints of the radius for the red clock.
- To stop plotting, press **EXIT**.

### To find the length of the radius:

- To see the coordinates of the endpoints, press **F3** (List). Enter these coordinates into the table.
- Press **MENU** and use     to highlight the RUN-MAT icon or press **1**. Use the distance formula,  $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ , and the coordinates to calculate the length of the radius for the red clock.



	X	Y	T
1	0.5	0.8	0
2	1.6	0.8	1
3			

0.5

AXTRNS EDIT DEL-BTM DEL-ALL SET

Math Rad Norm d/c Real

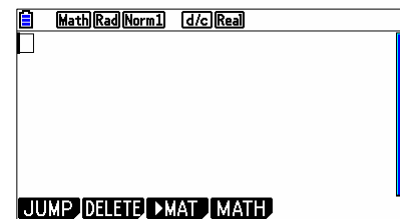
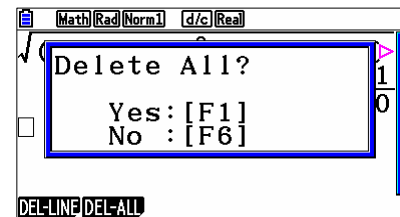
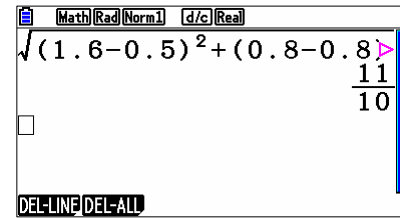
$$\sqrt{(1.6 - 0.5)^2 + (0.8 - 0.8)^2}$$

11  
10

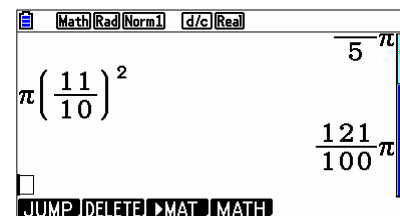
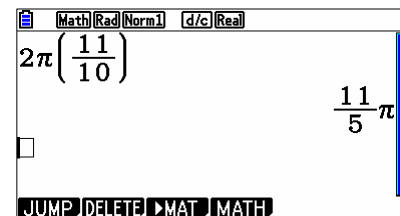
JUMP DELETE MAT MATH

**To find the circumference and area of the red clock:**

1. Press **F2** (DELETE) **F2** (DEL-ALL) **F1** (Yes) to clear the screen.



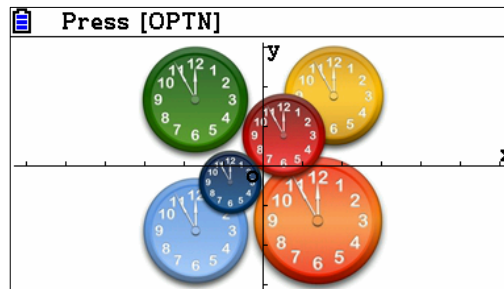
2. Using the formula  $C = 2\pi r$ , press **2** **SHIFT** **EXP** **(** **a/b** **1** **1** **▼** **1** **0** **▶** **)** **EXE** to calculate the circumference of the red clock. Enter the circumference into the table.
3. Using the formula  $A = \pi r^2$ , press **SHIFT** **EXP** **(** **a/b** **1** **1** **▼** **1** **0** **▶** **)** **x^2** **EXE** to calculate the area of the red clock. Enter the area into the table.



## Area of Similar Figures

## Activity

It seems logical that if one doubles the dimensions of an object then the area the object covers would also double; this is not the case. In this activity we will compare the perimeter and area of two similar objects when one dimension is altered.



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### Questions

- Find the coordinates of the endpoints of the radius for each of the clock faces and enter this into the table. Use the distance formula to find the length of each radius.

Color	$P_0$	$P_1$	Radius
Dark Blue			
Red			
Green			
Gold			
Orange			

2. Find the circumference for each clock, using  $C = 2\pi r$ , and enter this information into the table.

Color	Radius	Circumference
Dark Blue		
Red		
Green		
Gold		
Orange		

3. Divide the radius of the Red Clock by the radius of the Dark Blue Clock. Divide the circumference of the Red Clock by the circumference of the Dark Blue Clock. What do you notice?

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4. Divide the radius of the Gold Clock by the radius of the Orange Clock. Divide the circumference of the Gold Clock by the circumference of the Orange Clock. What do you notice?

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5. Does this hold true for another pair of clocks?

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6. Write a conjecture about the ratio of the radii of circles and the ratio of the corresponding circumferences.

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7. Find the area for each clock using  $A = \pi r^2$  and enter this information into the table.

Color	Radius	Area
Dark Blue		
Red		
Green		
Gold		
Orange		

8. Divide the radius of the Green Clock by the radius of the Red Clock. Divide the area of the Green Clock by the area of the Red Clock. What do you notice?

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9. Divide the radius of the Green Clock by the radius of the Gold Clock. Divide the area of the Green Clock by the area of the Gold Clock. What do you notice?

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10. Does this hold true for another pair of clocks?

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11. Write a conjecture about the ratio of the radii of circles and the ratio of the corresponding areas.

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**Extension**

1. Make a table for several squares using the side and perimeter. Does your conjecture for the circumference hold true for the perimeter of squares as well?  

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2. Make a table for several equilateral triangles using the side and area. Does your conjecture hold true for the area of equilateral triangles as well?  

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3. Since the ratio of length is equal to each other and the ratio of area is the square of the lengths, what do you think would happen with length and volume of regular figures?  

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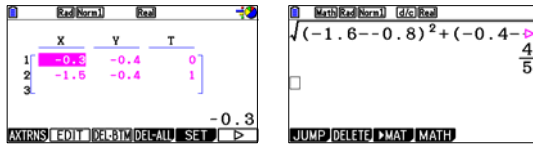
  

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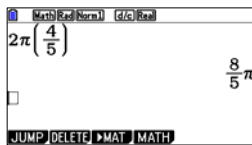
## Solutions

- Answers may vary according to the plotted points.  
[Screen Shots for Dark Blue Clock]



Color	P <sub>0</sub>	P <sub>1</sub>	Radius
Dark Blue	(-0.8, -0.4)	(-1.6, -0.4)	$\frac{4}{5}$
Red	(0.5, 0.8)	(1.6, 0.8)	$\frac{11}{10}$
Green	(-1.7, 1.7)	(-1.7, 0.4)	$\frac{13}{10}$
Gold	(1.8, 1.8)	(1.8, 3)	$\frac{6}{5}$
Orange	(1.4, -1.4)	(3, -1.4)	$\frac{8}{5}$

- Screen Shot for Dark Blue Clock



Color	Radius	Circumference
Dark Blue	$\frac{4}{5}$	$\frac{8}{5}\pi$
Red	$\frac{11}{10}$	$\frac{11}{5}\pi$
Green	$\frac{13}{10}$	$\frac{13}{5}\pi$
Gold	$\frac{6}{5}$	$\frac{12}{5}\pi$
Orange	$\frac{8}{5}$	$\frac{16}{5}\pi$

3. The ratios are equal.

Left screen:  $\frac{11}{10} \div \frac{4}{5} = \frac{11}{8}$

Right screen:  $\frac{11\pi}{10} \div \frac{8\pi}{5} = \frac{11}{8}$

4. The ratios are equal.

Left screen:  $\frac{8}{5} \div \frac{6}{5} = \frac{4}{3}$

Right screen:  $\frac{16\pi}{5} \div \frac{12\pi}{5} = \frac{4}{3}$

5. Yes
6. The ratio of the circumferences of two similar circles is equal to the ratio of their radii.
7. Screen Shot for Dark Blue Clock

Screen:  $\pi \left(\frac{4}{5}\right)^2 = \frac{16}{25}\pi$

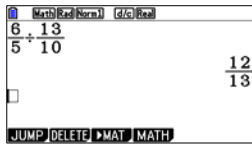
Color	Radius	Area
Dark Blue	$\frac{4}{5}$	$\frac{16}{25}\pi$
Red	$\frac{11}{10}$	$\frac{121}{100}\pi$
Green	$\frac{13}{10}$	$\frac{169}{100}\pi$
Gold	$\frac{6}{5}$	$\frac{36}{25}\pi$
Orange	$\frac{8}{5}$	$\frac{64}{25}\pi$

8. The ratio of the areas is the square of the ratio of the radii.

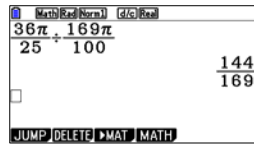
Left screen:  $\frac{13}{10} \div \frac{11}{10} = \frac{13}{11}$

Right screen:  $\frac{169\pi}{100} \div \frac{121\pi}{100} = \frac{13}{11}$

9. The ratio of the areas is the square of the ratio of the radii.



$$\frac{6}{5} \div \frac{13}{10} = \frac{12}{13}$$



$$\frac{36\pi}{25} \div \frac{169\pi}{100} = \frac{144}{169}$$

10. Yes
11. The ratio of the areas of two similar circles is equal to the square of the ratio of their radii.

### Extension Solutions

1. The tables will vary. The conjecture holds true.
2. The tables will vary. The conjecture holds true.
3. Since the first is equal and the second is the square, the third would be the cube of the ratio of the lengths.