Double Helix — Teacher Notes

Topic Area: Curve fitting; periodic functions; parametric equations

NCTM Standards:

- Recognize and apply mathematics in contexts outside of mathematics.
- Identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships.
- Use a variety of symbolic representations, including recursive and parametric equations, for functions and relations.

Objective

Given a photo of a screw, students will be able to fit a periodic function onto a section of the screw. Using their knowledge of trigonometric functions and how to restrict the domain of a function, students will create a regression based on plotting and analyzing a series of points. As an extension, students will define a model of the thread of a screw using parametric equations.

Getting Started

Have students work in small groups to help determine how to identify the amplitude, period, and phase shift of a sinusoidal equation.

Prior to using this activity:

- Students should understand how to graph parametric equations.
- Students should be able to determine an appropriate regression given a scatter plot.

Ways students can provide evidence of learning:

- Students should display graphs that match the photograph.
- Students should select appropriate regressions, given a scatter plot.

Common mistakes to be on the lookout for:

- Students may be careless in the placement of points.
- Students may have difficulty setting parameter values.

Definitions

- Three-dimensional helix

Formulas

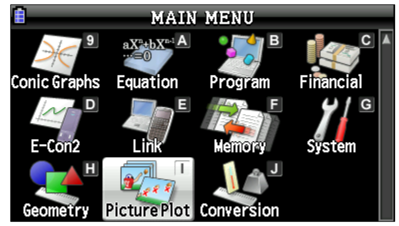
Parametric Helix Equations:

$$X_t = a \bullet \cos t$$
$$Y_t = a \bullet \sin t$$
$$Z_t = bt$$

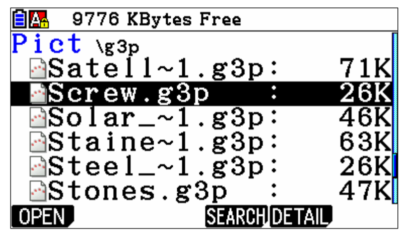
Double Helix "How To"

The following will walk you through the keystrokes and menus required to successfully complete the Double Helix activity.

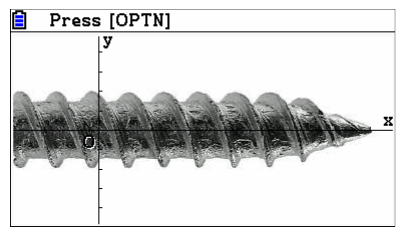
To open a background image in Picture Plot:



1. From the Main Menu, highlight the Picture Plot icon and press [EXE] or press [(].



2. Press [F1] (OPEN) to open the CASIO folder.



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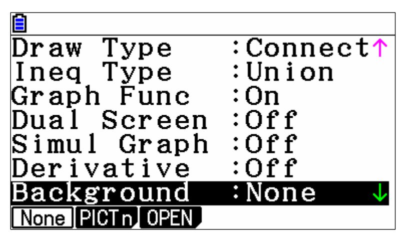
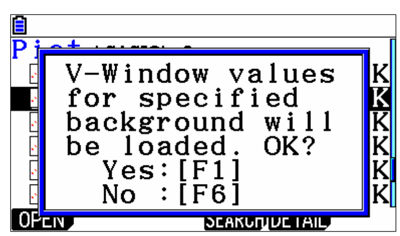
3. The g3p folder contains 47 background images. Press [▼] [F1] (OPEN) to open the folder. Scroll down the list of images and highlight the desired picture. You will be using the "Screw" image in this activity. Press [F1] (OPEN).

To add a background image to the Graph Menu:

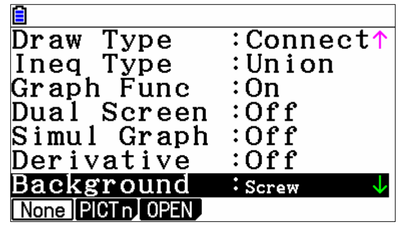


1. From within the GRAPH menu, press [SHIFT] [MENU] **(SET UP)** and [▼] until BACKGROUND is highlighted.



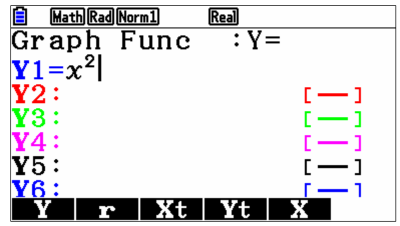
2. Press [F3] (OPEN), arrow down to the desired file and press [F1] (OPEN).

3. Press [F1] (YES) to accept the specified View Window.

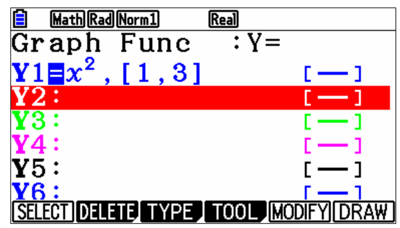


To restrict the domain of a function:

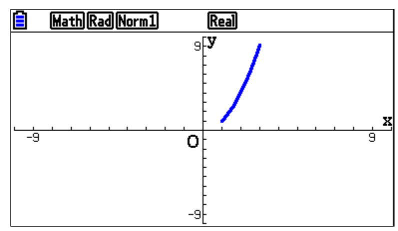
1. From within the GRAPH menu, enter the desired function, by pressing [X,θ,T] [x^2].



2. Domain restrictions are placed inside brackets. The restricted domain is [1, 3]. Immediately following the function, enter the following: [,] [SHIFT] [+] [1] [,] [3] [SHIFT] [−] [EXE].

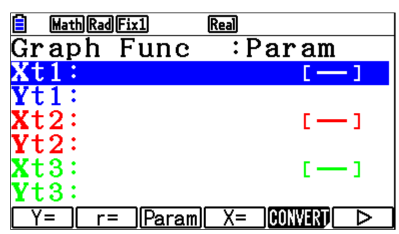


3. To see the restricted graph, press [F6] (DRAW).



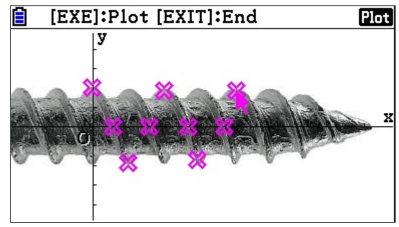
To set the equation line to receive parametric equations:

1. From within the GRAPH menu, go to an empty line and press [F3] (TYPE) [F3] (PARAM).



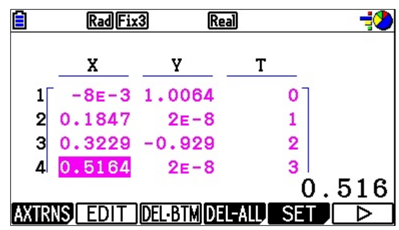
To plot points on the image and create a list of points:

1. To plot points, press [OPTN] [F2] (PLOT). A pink arrow will appear; use ◀ ▶ ▲ ▼ to move the arrow to where you would like for it to plot a point. (Any of the number keys can also be used to jump to different areas on the screen). Press [EXE] to plot the point.



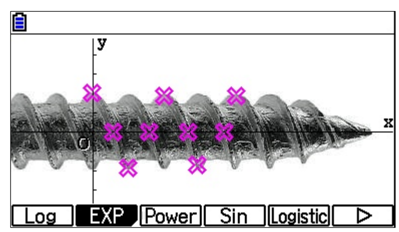
2. Continue moving the arrow and pressing [EXE] until you have all the desired points. Press [EXIT] to stop plotting.

3. Press [OPTN] [F3] (LIST) to view the list of points plotted. Press [EXIT] to go back to the image and points or press [F4] (DEL-ALL) to delete all points.

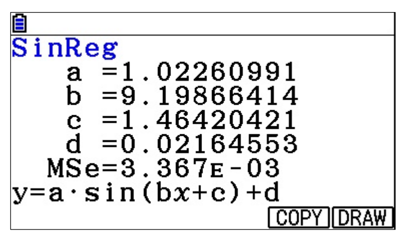


To perform a sinusoidal regression:

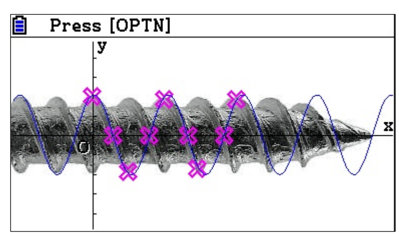
1. From within the Picture Plot icon, press [OPTN] [F6] (▷) [F2] (REG) [F6] (▷) [F4] (Sin).



2. From the regression screen, press [F5] (COPY) to record the regression information into a line in the Graph icon.

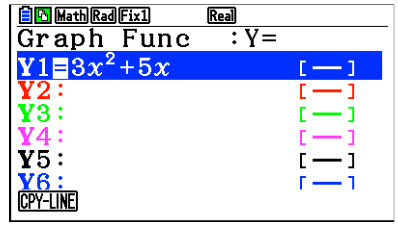


3. Press [F6] (DRAW) to sketch the regression through the plotted points.



To copy and paste a function to another location:

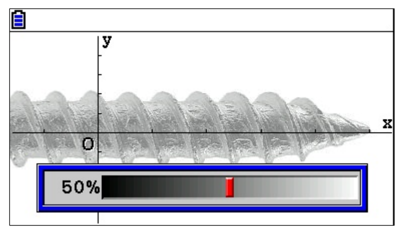
1. From within the Graph icon, highlight the desired function and press SHIFT 8 **(CLIP)** F1 (CPY-LINE) to copy.



2. Highlight the desired destination and press SHIFT 9 **(PASTE)** to paste.

To adjust the brightness of an image:

1. From within the Picture Plot icon, press OPTN F6 (▷) F6 (▷) F3 (FadeI/O).

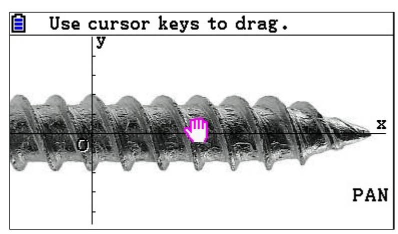


2. Use the replay buttons ◀ ▶ to lighten the image with each successive press.

3. Press EXE to accept the desired level of brightness.

To pan the axes over an image:

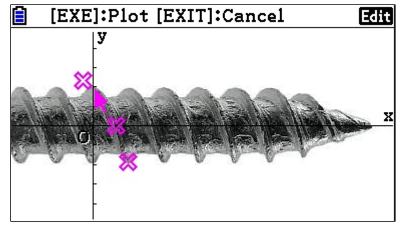
1. From within the Picture Plot icon, press OPTN F6 (▷) F5 (PAN) EXE to grab the axes.



2. Use the replay buttons ◀ ▶ ▲ ▼ to move the axes to the desired location.

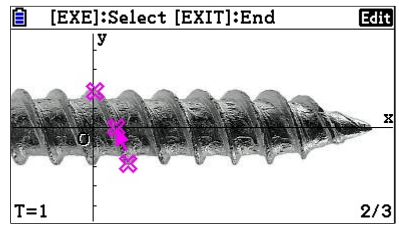
3. Press EXE to accept the desired movement of the axes.

To move a previously plotted point:



1. While the Picture Plot screen is displayed, press OPTN F6 (▷) F3 (EDIT).

2. Use replay buttons ◀ ▶ ▲ ▼ to move the pointer to the desired plot, then press EXE.



3. Use the cursor to move the pointer to the location to which you want to move the point, then press EXE.

4. After you have finished moving all the points you want, press EXIT.

Double Helix Activity

On April 25, 1953, the molecular biologists James D. Watson and Francis H. C. Crick published a pivotal paper in which they described the geometric shape of DNA, the molecule of life. The two biologists described the molecule as being in the form of a double helix — two helices that spiral around each other, connected by molecular bonds, to resemble nothing more than a rope ladder that has been repeatedly twisted along its length.

Given the neat way the two intertwined helices in DNA function in terms of genetic reproduction, you might think that the helix had important mathematical properties. Unfortunately, the equation of the helix is quite unremarkable and there seems to be relatively little to catch the mathematician's attention.

On a more productive note, helices are common in the world around us. Various sea creatures have helical shells and climbing vines wind around supports to trace out a helix. In the technological world of our own making, spiral staircases, corkscrews, drills, bedsprings, and telephone handset chords are helix-shaped. And what kind of a world would we have without the binding capacity the helix provides in the form of various kinds of bolts and screws.

In this activity, you will need to find a function to model the thread of a screw. You will accomplish this by plotting points along a screw’s thread and performing a regression on the plotted points. You will need to be able to adjust the plotted points to obtain a more accurate regression. A second regression will be drawn so that it is the reflection of the first regression. You will also need to determine an appropriate domain for each equation.

In the extension, you will need to plot appropriate points along features of a screw to determine various lengths. These lengths will be used as parameters for the equation for a helix projected in two-dimensions and will be drawn using parametric equations.

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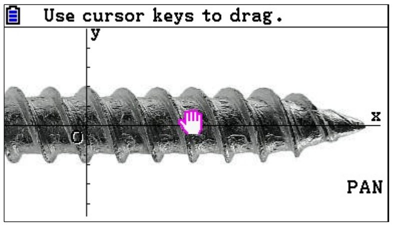
Questions

1. Pan the axes until the x-axis passes through the center of the screw and the y-axis passes through the apex of the third thread from the left. Record the resulting viewing window settings in the spaces provided and round each number to the nearest tenth.

Xmin	
Xmax	
Ymin	
Ymax	



2. Which regression on the Casio fx-CG series should be used to model the thread of the screw?

____

3. Notice that there are two separate spirals that form the thread of a screw. Plot at least 7 points along one spiral of the thread that follows the regression selected in Question 2. Round each coordinate to the nearest tenth and record the coordinates in the space provided

Point 1	
Point 2	
Point 3	
Point 4	
Point 5	
Point 6	
Point 7	
Point 8	
Point 9	
Point 10	

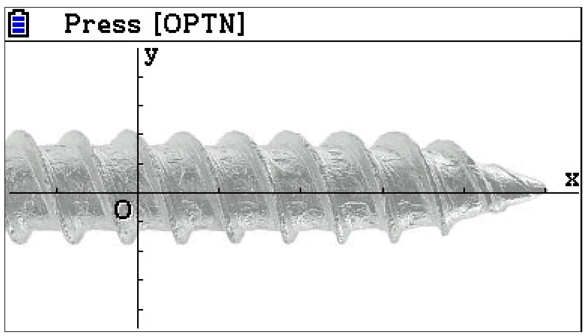
4. Perform the appropriate regression and write the regression equation below. Round each parameter to the nearest hundredth.

____

5. Draw the regression. Adjust plotted points as necessary to provide a tighter regression. If adjustments were made, rewrite the regression equation.

____

6. Show the results to your classmates and your teacher. Sketch the regression over the image below



7. Does the regression model the entire screw thread? If not, why do you think it does not? Explain.

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____

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8. Restrict the domain of the regression equation so that no more than two periods are displayed. What is the domain?

____

9. How could the second spiral be modeled without having to plot points, or performing another regression? Explain.

____

____

____

10. Write both equations for each spiral of the screw's thread.

____

____

Extension

The parametric model for a screw's thread is similar to the three dimensional helix. The two dimensional projection can be given by the equations:

$$Xt = T$$
$$Yt = R \bullet \sin\left[\frac{2\pi}{P}(T + w)\right]$$, where $R =$ radius of the screw, $P =$ distance between successive turns, $w =$ the phase shift

1. What is the radius of the screw?

____

2. What is the period (distance between successive turns) of the screw?

____

3. What is the phase shift (horizontal distance to correct for a standard sine function) of the screw?

____

4. The Picture Plot icon can only graph Y = f(x), therefore we must switch to the Graph icon, when graphing parametric equations. Write the parametric equation to model one spiral of the screw's thread, rounding all coefficients and constants to the nearest hundredth.

____

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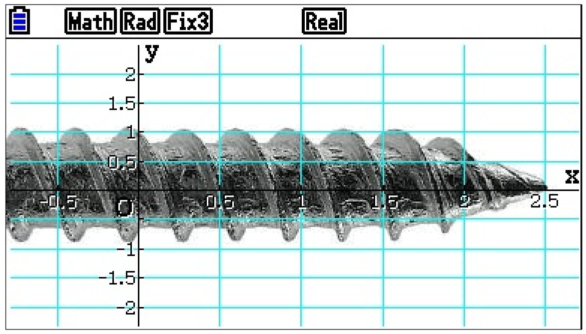
5. Graph the parametric function on the Casio fx-CG series in the Graph icon, with the appropriate values for $T\theta$, to create a model displaying at most two periods of the spiral. Show the results to your classmates and teacher. What values did you select for $T\theta$min and $T\theta$max?

____

6. Sketch the parametric function over the image below.

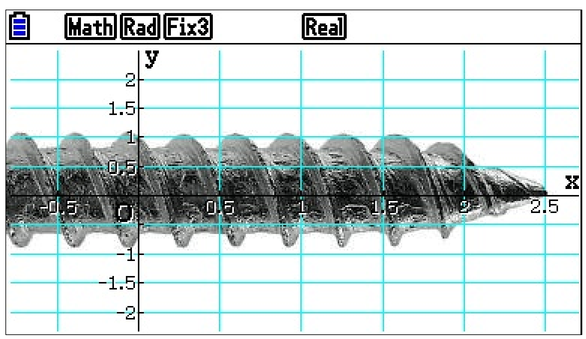


7. What is the *reflection* of the 1st parametric equation?

____

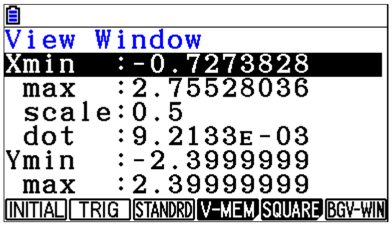
____

8. Sketch both parametric functions over the image below using appropriate values for Tθmin and Tθmax.



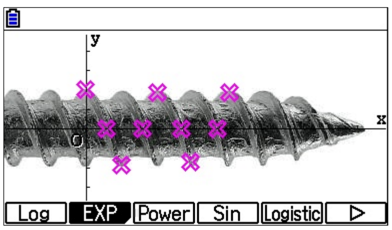
Solutions
Answers will vary, depending on the placement of the axes and plotted points.

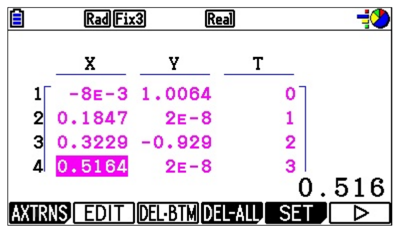
1. Xmin: –0.7 Xmax: 2.8 Ymin: –2.4 Ymax: 2.4



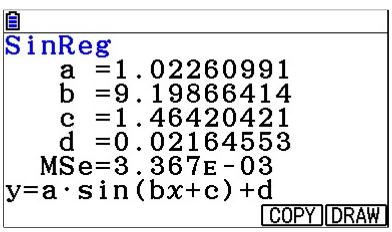
2. Sinusoidal

3.



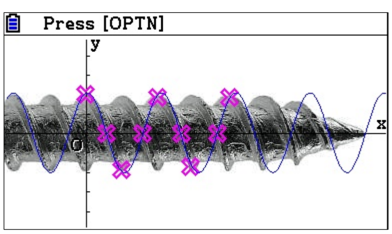


4. Regression equation: y = 1.02•sin(9.2x + 1.46) + 0.02



5. The adjustment is not necessary for this example.

6.

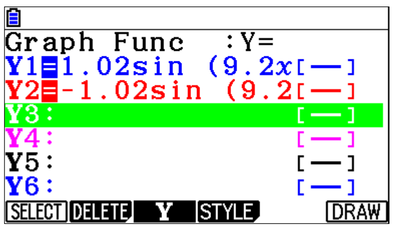


7. No. Possible reasons include, but are not limited to: (a) any regression is only valid over a very specific domain, (b) the radius of the screw is not constant, or (c) the entire screw’s thread is actually two spirals, not one spiral.

8. Answers may vary; one possible domain of regression equation is: [0, 1]

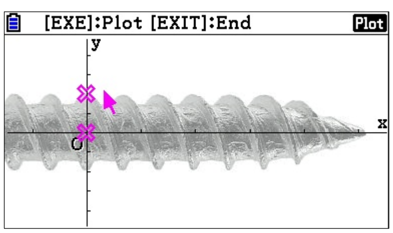
9. The second spiral equation is the reflection (over the x-axis) of the first spiral equation.

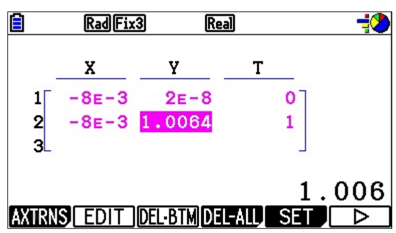
10. Y1 = 1.02 • sin(9.2x + 1.46) + 0.02
Y2 = –1.02 • sin(9.2x + 1.46) + 0.02



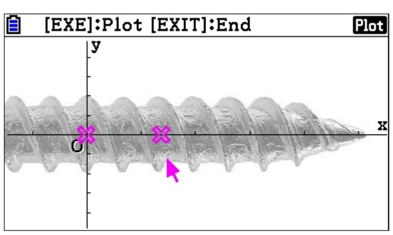
Extension Solutions
Answers will vary, depending on plotted points.

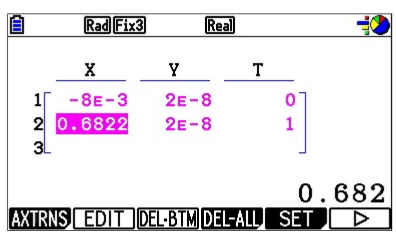
1. Radius = 1.01



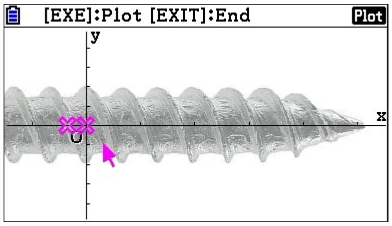


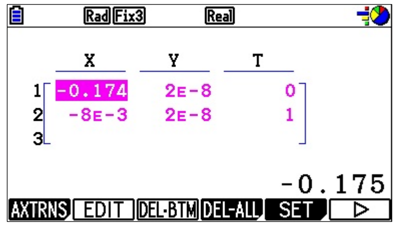
2. Period = 0.68





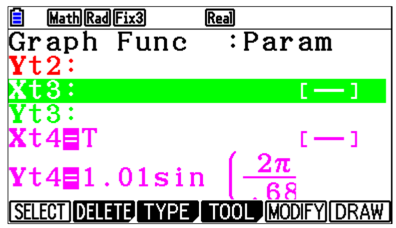
3. Phase Shift = 0.18





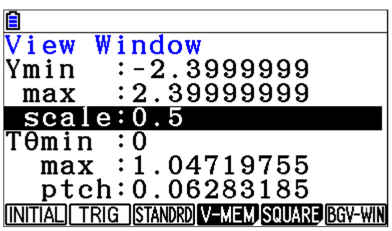
4. Xt = T

$$\text{Yt} = 1.01 \bullet \sin\left(\frac{2\pi}{0.68}(\text{T}+0.18)\right)$$

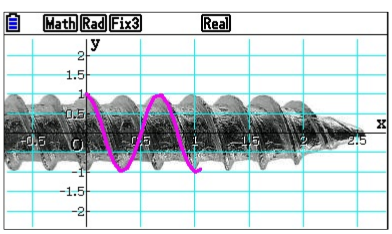


5. Tθmin = 0

Tθmax = $\frac{\pi}{3} \approx$ 1.0472

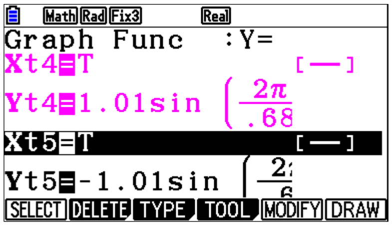


6.



7. Xt = T

Yt = –1.01•sin$\left(\frac{2\pi}{0.68}(T+0.18)\right)$



8.