Flying Fish Teacher Notes

Topic Area: Curve fitting quadratics to a video file

NCTM Standards:

- Create and use representations to organize, record, and communicate mathematical ideas.
- Use Mathematical models to represent and understand quantitative relationships.

Objective

Given a video file, students will be able to fit a quadratic to a goldfish jumping between bowls. Using their knowledge of quadratics, students will find and interrupt the meaning of the roots, maximum, and points that make up the curve.

Getting Started

Have students work in pairs to help determine what points should be plotted and to facilitate discussion of the questions.

Prior to using this activity:

- Students should have a basic understanding of what regression is and what it does.
- Students should understand the meaning of the roots of a parabola as they apply to real life situations.
- Students should be able to understand the meaning of the maximum of a parabola as it applies to a real life situation.
- Students should be able calculate an output value given an input value and calculate the x values given a y value, using quadratic formula.

Ways students can provide evidence of learning:

- Given a table of data, the student will be able to create an appropriate matrix to represent the data.
- Given two matrices, the student will be able to multiply the matrices and analyze the results.

Common mistakes to be on the lookout for:

- Students may be careless in the placement of points. Students should look for the center of the fish in every screen.
- Students may get confused when using X-cal and Y-cal; these are often interchanged by students.

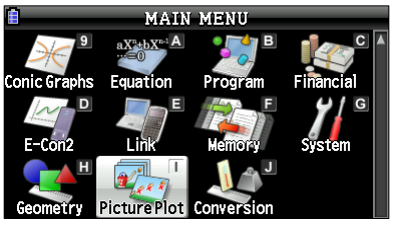
Definitions

- Quadratic
- Roots
- Maximum
- Regression

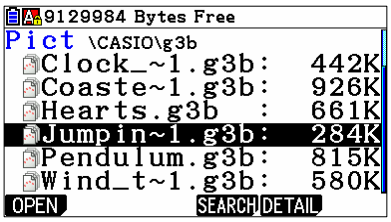
Flying Fish “How To”

The following will walk you through the keystrokes and menus required to successfully complete the flying fish activity.

=To open an image series file in Picture Plot:

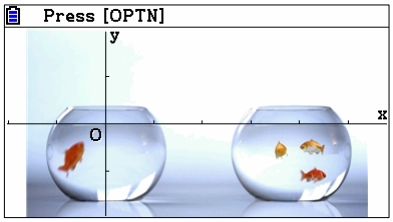


1. From the Main Menu, highlight the Picture Plot icon and press EXE or press (.

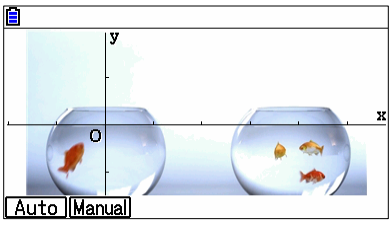


2. Press F1 (OPEN) to open the CASIO folder.

3. The g3b folder contains 8 image series files. Press F1 (OPEN) to open the folder. Scroll down the list of image series and highlight the desired image series. You will be using the “Jumpin~1” image series in this activity. Press F1 (OPEN).



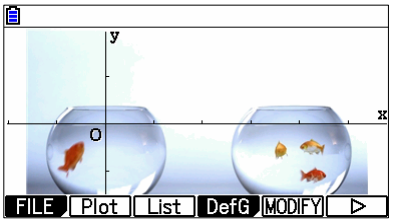
4. To preview the image series, press OPTN F6 (▷) F6 (▷) F2 (PLAY).
   The calculator gives you two options: F1 (Auto) or F2 (Manual). Auto plays through the entire sequence of images; Manual requires you to press ◀ or ▶ to move through each image.

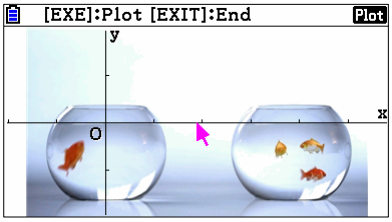


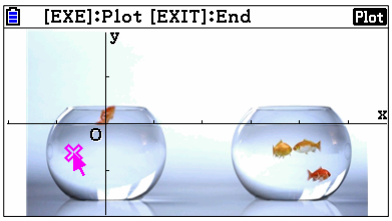
5. Press AC/ON to stop the image series.

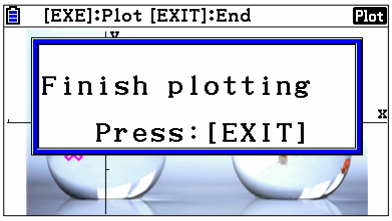
To plot points on the image series and create a list of points:

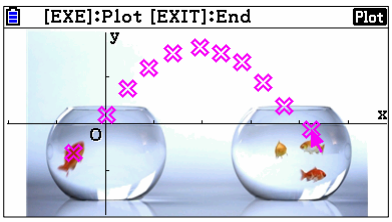
1. To plot points, press OPTN F2 (PLOT).
   A pink arrow will appear; use ◀ ▶ ▲ ▼ to move the arrow to where you would like for it to plot a point. (Any of the number keys can also be used to jump to different areas on the screen).
   Press EXE to plot the point and advance the image series one frame.
2. Continue moving the arrow and pressing EXE until the image series ends. To stop plotting before the end of the image series, press EXIT.





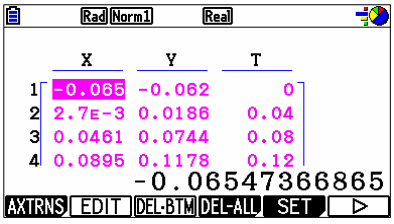






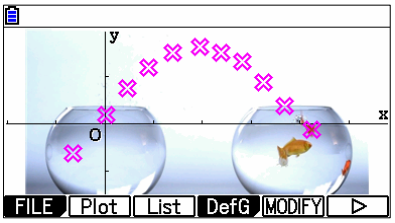
To view the list of data points:

1. Press F3 (List) to view the list of points plotted.
   Press EXIT to go back to the picture and points

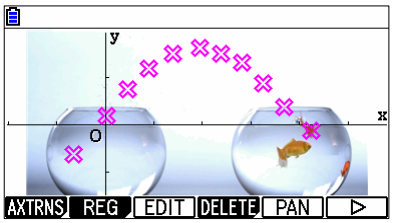


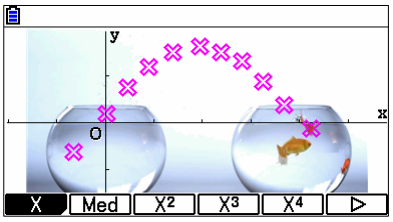
To create a best fit line or curve of best fit:

1. Press F6 (▷) F2 (REG).

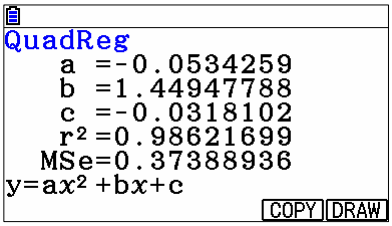


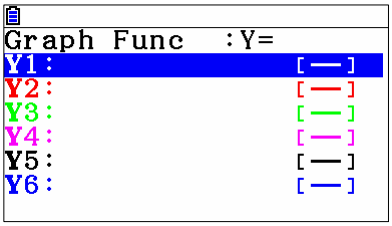
2. Choose the appropriate regression model.
   In this case, it will be X^2, since it is a parabolic Motion, so press F3.



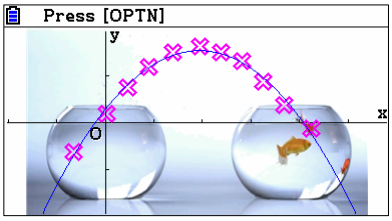


3. Press F5 (Copy) and EXE to copy the equation to the Graph menu.

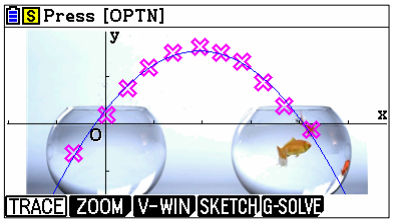




4. Press F6 (DRAW) to see the regression curve and the points.



To find roots, Max, and X and Y values:



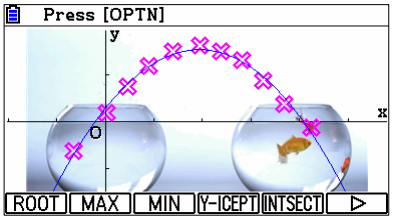
1. Press SHIFT F5 **(G-Solv)** to analyze features of the regression.

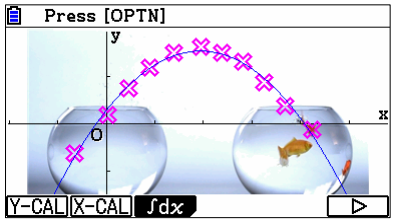
   F1 (ROOT)- finds the root(s). The left-most root will always display first. Press ▶ to see the next root to the right.

   F2 (MAX)- determines the maximum.

   F6 (▷) F1 (Y-CAL)- finds a y-value given x.

   F2 (X-CAL)- finds an x-value given y.





Flying Fish Activity

Questions

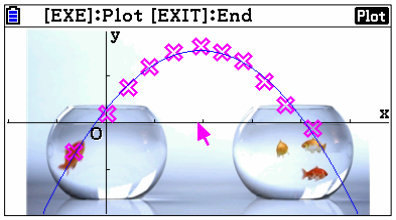
1. Play the image series and write down one question you would like to answer in this activity.

_____

_____

2. Plot points on the image series and write down the list of values below. Your picture should look similar to the one below.

Horizontal	Vertical



3. Which data points represent the roots of the parabola created by the plot?

_____

4. According to the data, what is the maximum height achieved by the fish?

_____

5. What does the first data point represent?

_____

6. Use the data set to find a best fit quadratic equation. What is the equation (round to the nearest hundredth)?

_____

7. What are the roots of the quadratic equation? Think about the situation. Do the x-coordinates make sense? Explain.

_____

_____

_____

8. What should they be? Why?

_____

_____

9. If the second bowl was placed on the floor 20 units down, how far from the x- axis should it be placed?

_____

10. If you would like to train the fish to jump through hoops and you place the hoops at a height of 9 units up, how far along the x axis should they be placed?

_____

11. Would the fish be able to jump over a bar placed at a height of 15 units? What is the highest the fish should be able to jump?

_____

12. I want to place the second bowl at x = 100 how far down must it be placed?

_____

13. The fish appears to be 6 units long. The world record high jump is 8 feet, by Javier Sotomayor Sanabria of Cuba; he is 6 foot 5 inches tall. Relative to height, who is jumping higher? Explain you answer.

_____

_____

14. By how much?

_____

15. Who is a better long-jumper, the fish or Mike Powell, who is 6 foot 1 inch and jumped 29 feet 4.5 inches?

_____

Extensions

1. Write the equation in vertex form and find the coordinates of the focus and axis of symmetry. What do these values represent in terms of the fish?

_____

_____

2. Show that the distance from any point to the focus is equal to the distance from that point to the directrix.

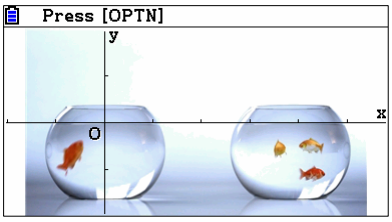
_____

_____

3. Find the equation of a line for a cat that snatches the fish out of midair at the maximum point, if the cat starts at the point (-12, -14).

_____

4. Find the equation of the hyperbola created by the inner curves of the bowl.

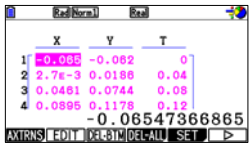


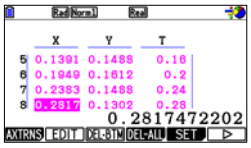
_____

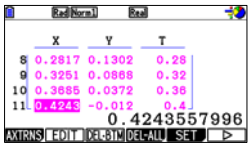
Solutions

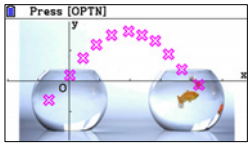
1. Answers will vary.
   Will the fish make it?
   How high does it go?
   How far does it jump?

2. Answers will vary, depending on points plotted.
   Hint: students should try to plot points through the thickest part of the fish in every frame (center of gravity).



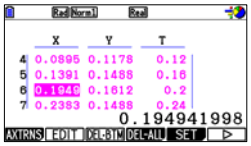






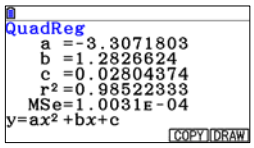
3. The second data point and the second to last data point. The fish is underwater on the first and last data point.

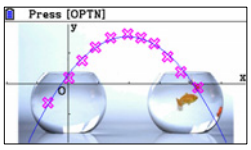
4. Answers will vary, depending on the plotted points.
   Approximately (14.857, 10.052)



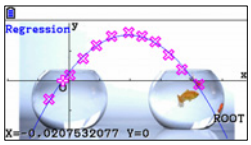
5. The initial take off point or launch point.

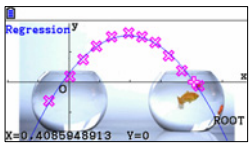
6. Answers will vary, depending on the points plotted.
   $y = -0.053x^2 + 1.45x - 0.3$





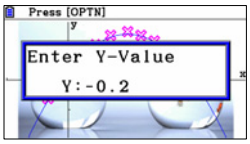
7. Roots (0.02, 0) and (27.11, 0).

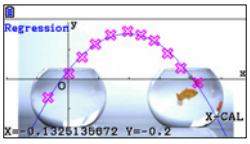


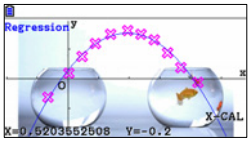


8. The first root should be (0, 0) the fish is jumping through the origin. The second root may be affected by the distortion of the fish landing in the water. The splash is behind the fish.

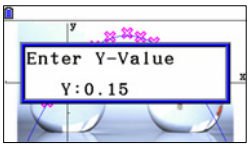
9. The two answers the calculator gives are (-10.05, -20) and (37.18, -20). The first answer does not make sense, so the second answer is the solution; 37.18 units over on the x-axis.

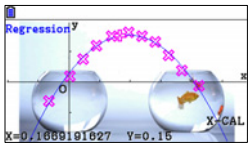


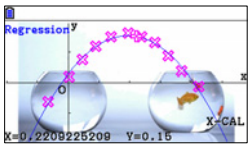




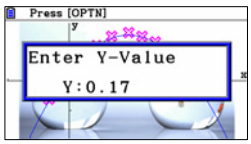
10. Placed the hoops at 9.70 and 17.43 units on the x-axis.

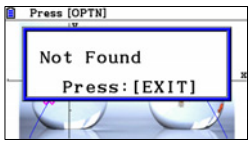


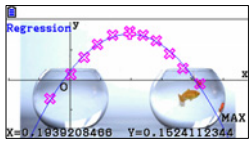




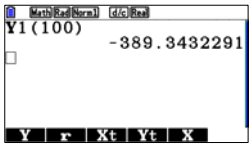
11. The fish will not make a jump of 15 units high. The calculator will give a “Not Found” message. The highest predicted jump is the maximum of the curve or 13.57.



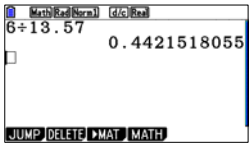


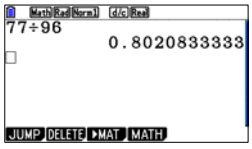


12. Students will need to plug in a value of 100 by hand or change the maximum x value to be over 100 on the screen. From the RUN-MAT icon, enter
VARS F4 F1 1 (1 0 0) EXE to get a solution of -389.34 units.



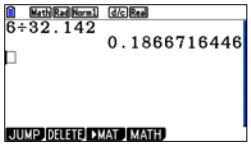
13. The height to jump ratio for the fish is 0.44 and the record height to jump ratio is 0.80. Javier, the record holder, is jumping higher.

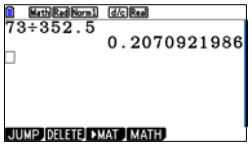




14. Javier jumped 0.36 times as far as the fish.

15. The fish jumped a distance of 32.142 units, so the height to jump ratio is 0.19. The world record ratio is 0.21; so Mike is a better long jumper.

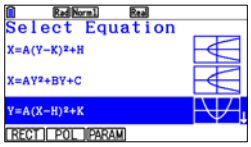


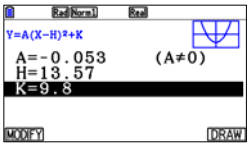


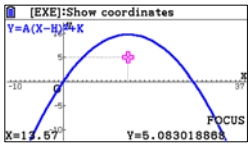
Extension Solutions

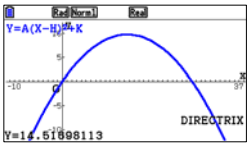
1. Answers will vary, depending on points plotted.
If we use $y = -0.053x^2 + 1.45x - 0.3$, the answer should be
$y = -0.053(x - 13.57)^2 + 9.80$.

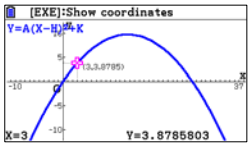
2. The focus is at (13.57, 5.08), the directrix is at y = 14.52, a point on the curve is (3, 3.879). Using the distance formula, the distances to the focus is 10.638 and the distance to the directrix is 10.641.

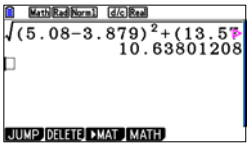


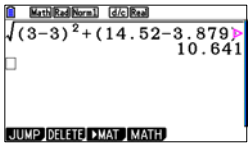




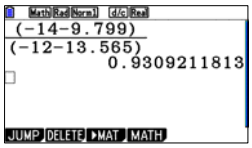


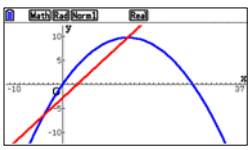






3. The maximum point is (13.565, 9.799). Using the slope formula, the slope of the line between the cat's starting point and the maximum point is 0.931. In point-slope form, the equation would be y + 14 = 0.931(x + 12). In slope-intercept form, the equation would be y = 0.931x – 2.828.





4. $\dfrac{(x-14)^2}{16}-\dfrac{(y+4)^2}{16}=1$