

Volume of Rotated Figures

Teacher Notes

Topic Area: Volume of Solids by Shell Method

NCTM Standards:

- Use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume;
- Use Cartesian coordinates and other coordinate systems, such as navigational, polar, or spherical systems, to analyze geometric situations;

Objectives

Given a photo file, students will be able to fit an equation onto the picture and find the equation for the line of best fit. Using their knowledge of polynomial functions and integral calculus, the students will find the volume of a figure formed by rotating a polynomial around a specified axis.

Getting Started

Have students work in pairs to determine the coordinates of points used to find the line of best fit for a polynomial and use this equation to solve problems involving rotation of solids.

Prior to using this activity:

- Students should have an understanding of how to find a regression line.
- Students should have an understanding of polynomial functions.
- Students should be able to write and solve an integral given the function with upper and lower bounds.

Ways students can provide evidence of learning:

- Students will be able to use their knowledge of geometry to determine upper and lower bounds for an integral.
- Students will be able to find the volume of a figure using the shell method.

Common mistakes to be on the lookout for:

- Students may have difficulty determining upper and lower bounds.
- Students may have difficulty with the integration of a polynomial.
- Students may use the wrong polynomial for the integration.

Definitions:

- Regression
- Upper and Lower Bounds
- Integral

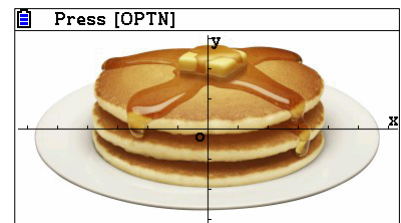
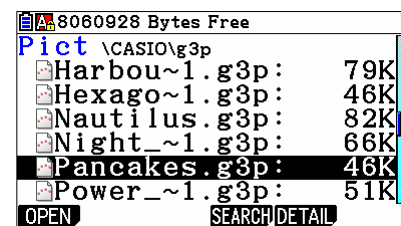
Volume of Rotated Figures

“How To”

The following will walk you through the keystrokes and menus required to successfully complete the volume of rotated figures activity.

To open a background image in Picture Plot:

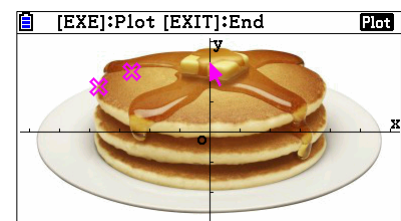
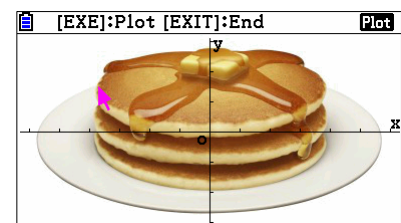
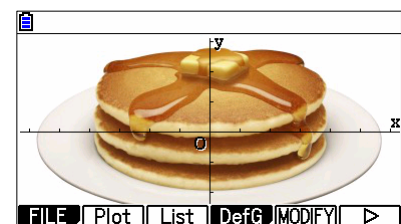
1. From the Main Menu, highlight the Picture Plot icon and press **EXE** or press **□**.
2. Press **F1** (OPEN) to open the CASIO folder.
3. The g3p folder contains 47 background images. Press **▼** **F1** (OPEN) to open the folder. Scroll down the list of images and highlight the desired image. You will be using the “Pancakes” in this activity. Press **F1** (OPEN).



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To plots points on the image and create a list of points:

1. The status bar at the top of the screen displays what operations you have to choose from; for this image, you will need to press **OPTN**.
2. To plot points on the image, press **F2** (Plot). A pink arrow will appear. Use **◀** **▶** **▲** **▼** to move the arrow where you would like to plot a point. (Any number of keys can also be used to jump to different areas on the screen). Press **EXE** to plot a point on the picture.



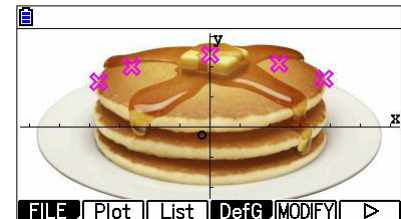
To view the list of data points you selected:

1. Press **OPTN** **F3** to view the list of points plotted. Press **EXIT** to go back to the image and points.

	X	Y	T
1	-3.7	1.5	0
2	-2.6	2	1
3	0	2.4	2
4	2.3	2.1	3

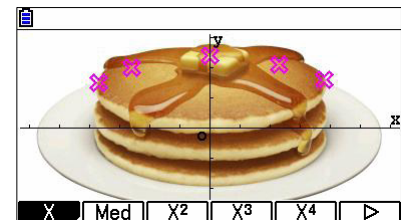
-3.7

AXTRNS EDIT DEL-BTM DEL-ALL SET



To create a line of best fit:

1. Press **F6** (\triangleright) and **F2** (REG).
2. In this case the regression line is a quadratic, therefore press **F3** (X^2).
3. Press **F5** (Copy) and **EXE** to copy the equation into the Graph Menu. .
4. Press **F6** (DRAW) to see the regression line generated by your selected points.



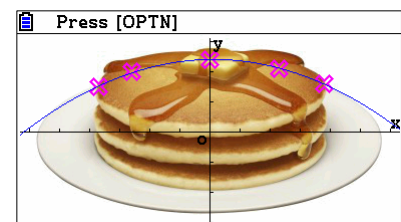
QuadReg

a = -0.0605453
b = 0.01417055
c = 2.40710927
 $r^2 = 0.99490182$
MSe = $1.3968E-03$
 $y = ax^2 + bx + c$

COPY DRAW

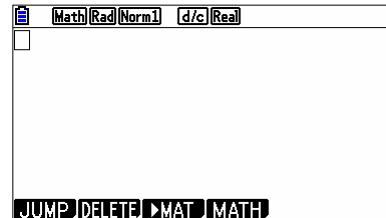
Graph Func : Y=

Y1: [—]
Y2: [—]
Y3: [—]
Y4: [—]
Y5: [—]
Y6: [—]

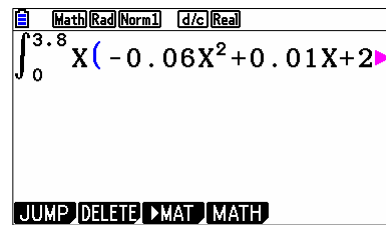
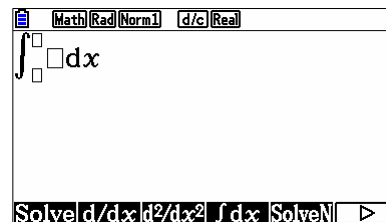


To find the volume of the pancakes by the shell method:

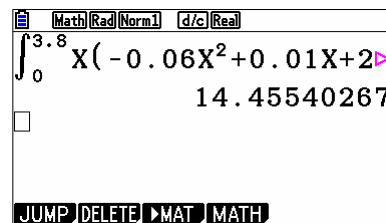
1. Press **MENU**, highlight the RUN•MATRIX icon and press **EXE**.



2. Press **OPTN** **F4** (CALC) **F4** ($\int dx$). Using the formula for finding the volume of an area rotated about the y-axis, enter this into the calculator.



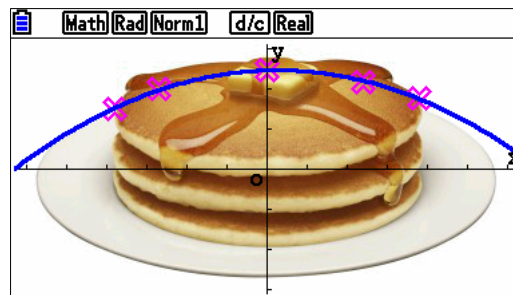
3. Press **EXE** to see the volume.



Volume of Rotated Figures

Activity

People are starting to think more about their diet and what they eat. To some, a pancake breakfast is a great way to start a Saturday or Sunday morning. However, these delicious breakfast icons are made from flour which is considered high in calories, especially if made with processed or white flour. In the picture below you will see a stack of three pancakes. You will be estimating the number of calories based on the volume of a stack of pancakes.



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1. List the coordinates for the curve of the top of the pancake.

x-coordinate	y-coordinate
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.

2. What is the regression line for the curve across the top of the pancakes?

3. In terms of the radius, what is the range for the radius of the pancake (assuming each unit on the graph is 1 inch)?

4. The volume of a solid formed by rotating an area about the y-axis is

$$V = 2\pi \int_a^b p h \, dx, \text{ where } p \text{ is the radius and } h \text{ is the height.}$$

Write the integral to find the volume of the stack of pancakes using the regression line as the height and the range for the radius as the lower and upper bounds.

5. Find the volume of the stack of pancakes.
-

6. For these pancakes, 1 oz. $\approx .55 \text{ in.}^3$ and 8 oz. = 453 calories. Find the number of calories (without the butter and syrup) for the stack of pancakes.
-

Extension

1. If the pancakes had a radius of 2 in., what would be the volume of the stack of pancakes?
-

2. What was the change in volume of the pancakes?
-

3. What would be the volume and calorie count for a single pancake?

a. Volume: _____

b. Calorie Count: _____

Solutions:

1.

	X	Y	T
1	-3.8	1.5	0
2	-2.7	2	1
3	0	2.4666	2
4	2.4	2.1666	3

-3.8

AXTRNS EDIT DEL-BTM DEL-ALL SET ▶

	X	Y	T
3	0	2.4666	2
4	2.4	2.1666	3
5	3.8	1.7666	4
6			

AXTRNS EDIT DEL-BTM DEL-ALL SET ▶

x-coordinate	y-coordinate
1. -3.8	1. 1.5
2. -2.7	2. 2.0
3. 0	3. 2.6
4. 2.4	4. 2.2
5. 3.8	5. 1.8

2.

	X	Y	T
1	-3.8	1.5	0
2	-2.7	2	1
3	0	2.4666	2
4	2.4	2.1666	3

-3.8

X Med X² X³ X⁴ ▶

QuadReg

a = -0.0572998
b = 0.02900764
c = 2.46225684
r² = 0.99366964
MSe = 1.7345E-03
y = ax² + bx + c

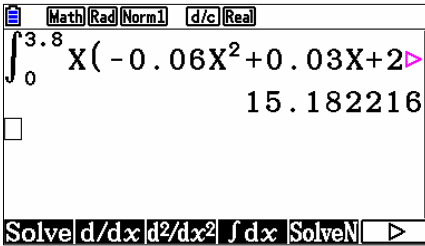
COPY DRAW

Press [OPTN]

$$y = -0.06x^2 + 0.03x + 2.46$$

3. $0 < x < 3.8$ in.

4.
$$V = 2\pi \int_0^{3.8} x(-0.06x^2 + 0.03x + 2.46)$$

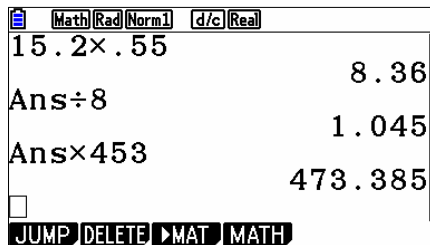
5. 

$V = 15.2 \text{ in.}^3$

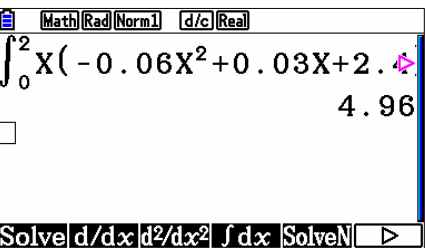
6. $15.2 \text{ in.}^3 = 8.36 \text{ oz.}$

$8.36 \div 8 = 1.045$ (No. of 8 oz.)

$1.045 \times 453 \text{ Cal. per 8 oz.} = 473 \text{ Cal.}$



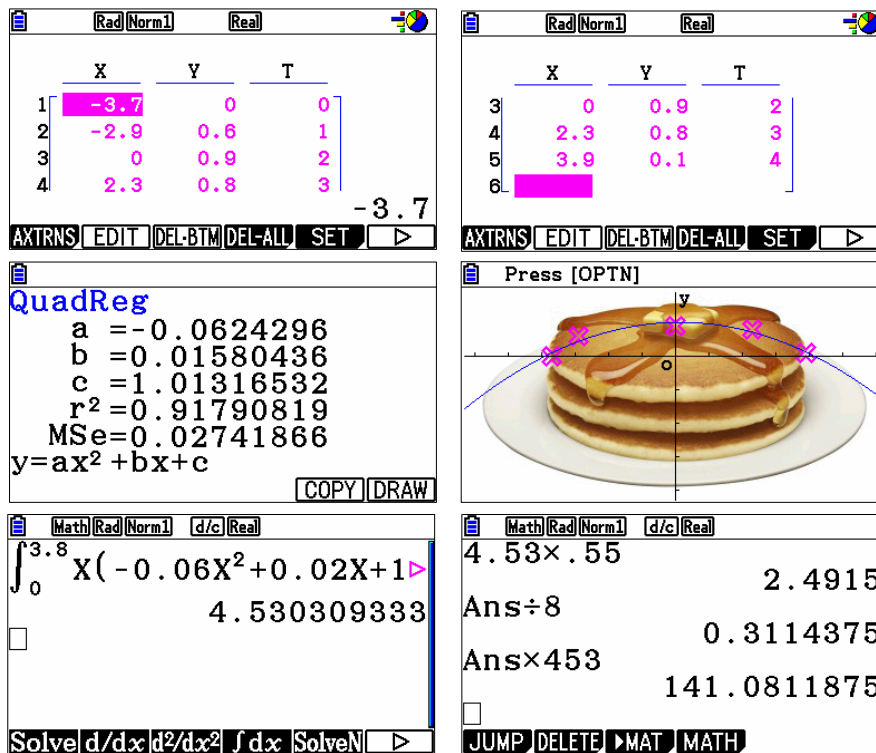
Extensions

1. 

$V = 4.96 \text{ in.}^3$

2. $8.36 - 4.96 = 3.4 \text{ in.}^3$

3.



$$V = 4.53 \text{ in.}^3$$

$$\text{Calorie Count} = 141 \text{ Cal.}$$