Arithmetic and Geometric Sequences on the CASIO ClassPad **Name:**________________________________

A **sequence** is a list of numbers. For example: 1, 3, 5, 7, 9, 11, . . . is a sequence.

Each time you work with a list of numbers, you are working with a sequence. The notation used to indicate a sequence is a_n.

For example: a_n = 1, 3, 5, 7, 9, 11, . .

a_2 would be the 2nd term in the sequence, a_{100} would be the 100th term, a_n would be the nth term and a_{n+1} would be the term following the nth term!

If a sequence has a pattern, one way to describe it is with a **recursive formula**. A **recursive formula** uses a previous term to get the next term.

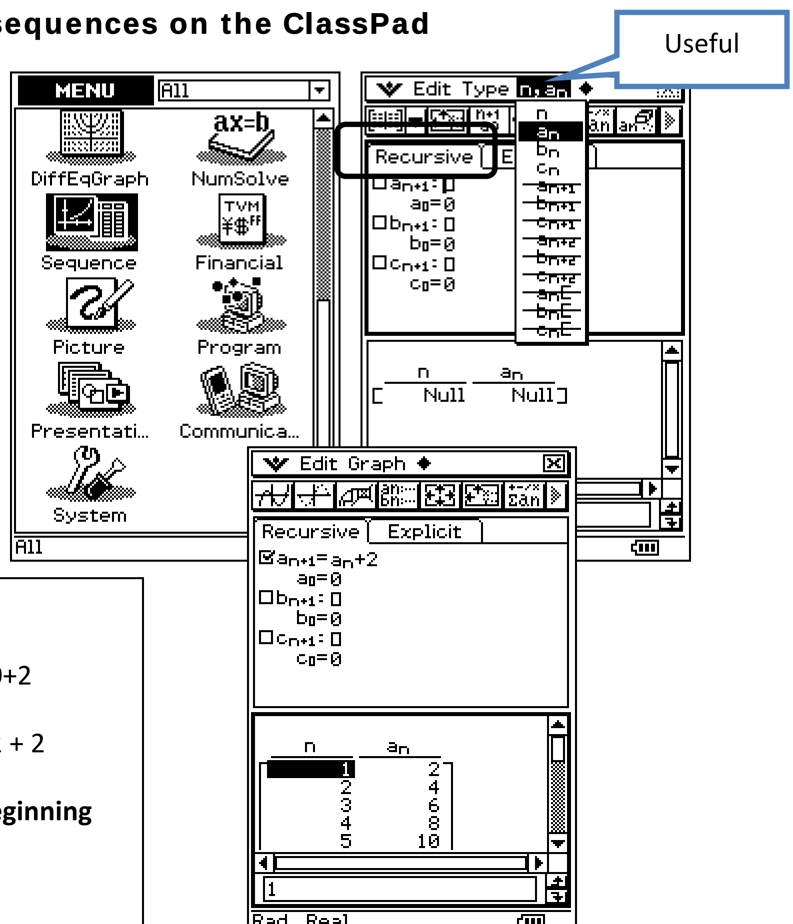
For example: $a_n = a_{n-1} + 2$, given $a_1 = 1$

Using this formula, we can find the value of any term if we know the previous term:

$$a_2 = a_1 + 2$$
$$= 1+2 \quad \text{(Recursively using } a_1 \text{ to get } a_2\text{)}$$

Exploring recursive formulas for sequences on the ClassPad

a. Tap m and then H
b. Input $a_{n+1} = a_n + 2$ and press E
c. Keep $a_0 = 0$
d. Tap the 1st toolbar button (#)
e. You can increase the table settings by tapping 8



Understanding the formula:

For n=1, $a_{n+1} = a_n + 2$ becomes $a_2 = a_1 + 2 = 0+2$

For n=2, $a_{n+1} = a_n + 2$ becomes $a_3 = a_2 + 2 = 2 + 2$

List the numbers in this sequence list form beginning with n=0:

{0, 2, 4, 6, 8, 10, …}

❖ Exercise 1:

Complete the following table. You can edit the formula already in the ClassPad to save time. However, remember to re-generate the table by tapping the # icon.

Recursive Formula	Sequence as List	Pattern
$a_{n+1} = a_n + 2,\ a_0 = 0$	{0, 2, 4, 6, 8, 10, …}	Consecutive terms differ by 2
$a_{n+1} = a_n + 2,\ a_0 = 1$		
$a_{n+1} = a_n + 3,\ a_0 = 1$		
$a_{n+1} = a_n + 4,\ a_0 = 0$		

Definitions for Arithmetic Sequence Formulas

An <u>Arithmetic</u> sequence is a sequence that has a common difference between any two consecutive terms. Another way to think of this is if you are adding the same number to each term to complete the sequence, it is an arithmetic sequence.

The **<u>recursive form</u>** of an Arithmetic Sequence is: $a_n = a_{n-1} + d$ or $a_{n+1} = a_n + d$

Where **d is the **common difference** between any two consecutive terms.

The **<u>explicit form</u>**, sometimes called the nth-term formula, for an Arithmetic sequence is given by: $a_n = a_1 + (n-1)d$ for $n > 1$
(Note: We need to know the value of the first term)

Side note:

Consider how the <u>explicit form</u> is derived by considering the following expansions of the recursive form:

$a_2 = a_1 + d$

$a_3 = a_2 + d = (a_1 + d) + d = a_1 + 2d$

$a_4 = a_3 + d = (a_2 + d) + d = ((a_1 + d) + d) + d = a_1 + 3d$

...

$a_n = a_1 + (n-1)*d$

Example:

Find the explicit form of an arithmetic sequence given the first term is 3 and the tenth term is 30.

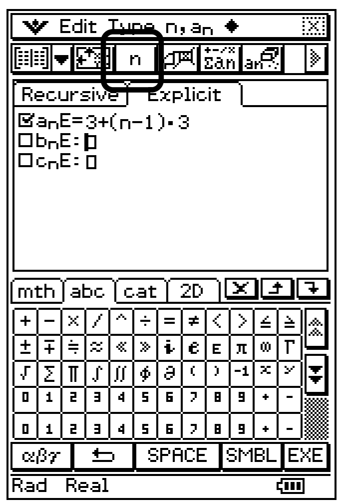
Given:	**Know:**	**Need:**
$a_1 = 3$ and $a_{10} = 30$	$a_n = a_1 + (n-1)*d$	Value of d

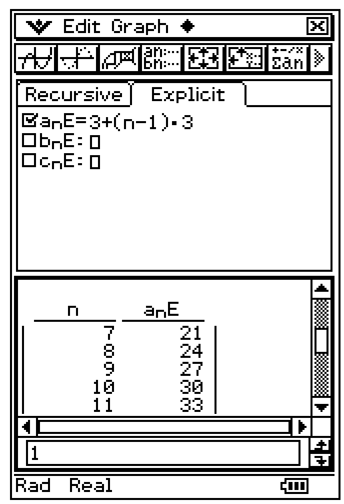
Substitute to find d!!

$$a_n = a_1 + (n-1)*d$$
$$a_{10} = a_1 + (10-1)*d$$
$$30 = 3 + 9*d$$
$$d = 3 \rightarrow a_n = a_1 + (n-1)*3$$

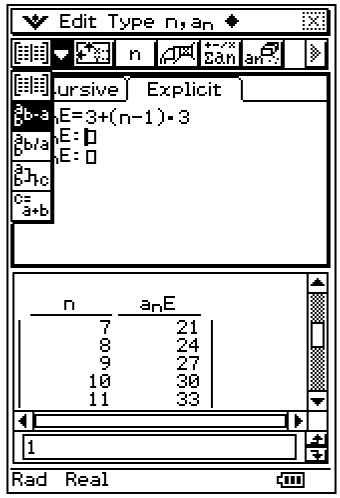
Checking your result on the ClassPad:

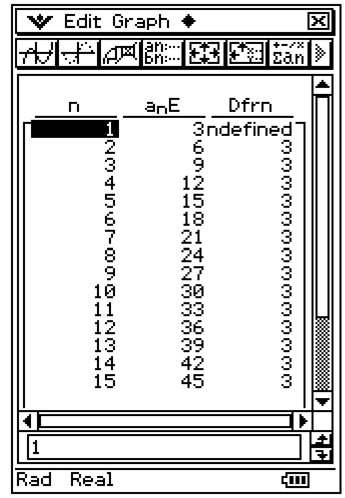
a. Tap m and then H
b. Tap the **Explicit tab**
c. Input $a_{nE} = 3 + (n-1)x3$
d. Press E
e. Tap the 1st toolbar button (#)
f. Check: for n = 1, does $a_n = 3$?
g. Check: for n = 10, does $a_n = 30$?





a. **New!!** Great feature!
b. Tap back in the upper window to give it focus
c. Tap the n next to the table button
d. Select the difference button (b-a)
e. Since the difference is a constant, we know we have an Arithmetic sequence.





❖ Exercise 2

a) Find the explicit form for the Arithmetic sequence given the first term is -5 and the forth term is 7.

b) Check your answer on the ClassPad and find the value of the 100th term.

❖ Exercise 3:

Complete the following table using the recursive tab. You can edit the formula already in the ClassPad to save time. However, remember to re-generate the table by tapping the icon.

Recursive Formula	**Sequence as List**	**Pattern**
$a_{n+1} = 2a_n, a_0 = 1$	{2, 4, 8,16, 32, ...}	Quotient of consecutive terms is 2 (4/2=2, 8/4=2...)
$a_{n+1} = 3a_n,\ a_0 = 2$		
$a_{n+1} = .5a_n,\ a_0 = 20$		

Definitions for Geometric Sequence Formulas

A <u>Geometric</u> sequence is a sequence that has a common ratio between any two consecutive terms ($a_{n+1}/\ a_n$ = constant).

The **<u>recursive form</u>** of a Geometric Sequence is: $a_n = r \cdot a_{n-1}$ or $a_{n+1} = r \cdot a_n$

Where **r is the **common ratio** between any two consecutive terms ($a_{n+1}/ a_n = r$).

The **<u>explicit form</u>**, sometimes called the nth-term formula, for a Geometric sequence is given by: $a_n = a_1 \cdot r^{n-1}$ for $n > 1$ (Exponent is a positive integer)

Side note:
Consider how the <u>explicit form</u> is derived by considering the following expansions of the recursive form:

$a_2 = a_1 \times r$

$a_3 = a_2 \times r = (a_1 \times r) \times r = a_1 \times r^2$

$a_4 = a_3 \times r = (a_1 \times r^2) \times r = a_1 \times r^3$

...

$a_n = a_1 \times r^{n-1}$

❖ Exercise 4

a) Find the <u>recursive</u> form for a Geometric sequence given $a_0 = 3$ and the $r = 2$. List the first five terms of the sequence. Check using the ClassPad's table option.

b) Find the <u>explicit</u> form for a Geometric sequence given the first term is 6 and the $r = 1/2$. List the first five terms of the sequence. Check using the ClassPad!

c) Complete the following table:

Sequence	**Arithmetic, Geometric or Neither?**	**Recursive Formula**	**Explicit Formula**
{3,6,9,12,15,...}	Arithmetic ($d=3$)	$a_n = a_{n-1} + 3$, $a_0 = 3$	$a_n = 3 + (n-1)x3$
{2,4,8,16,32,...}	Geometric ($r=2$)	$a_n = 2a_{n-1}$, $a_0 = 2$	$a_n = 2x2^n$

{3,9,27,81,243,...}			
{333,111,37,...}			
{4,9,14,19,24,...}			
{1,4,5,8,9,12,13,...}			
{-5, -2, 1, 4, 7, ...}			