Introduction to Linear Programming using the ClassPad

Name: _____

What is linear programming?

Linear programming is a way of finding the optimal conditions to maximize or minimize a quantity using linear inequalities (called the constraints). We begin by defining multiple inequalities (constraints) for the situation given. Next, each inequality is graphed to find the region of interest (called the feasibility region) and the points of intersection (vertices) are identified. Each vertex point is substituted into our "objective function" to find the optimal solution!

Example 1

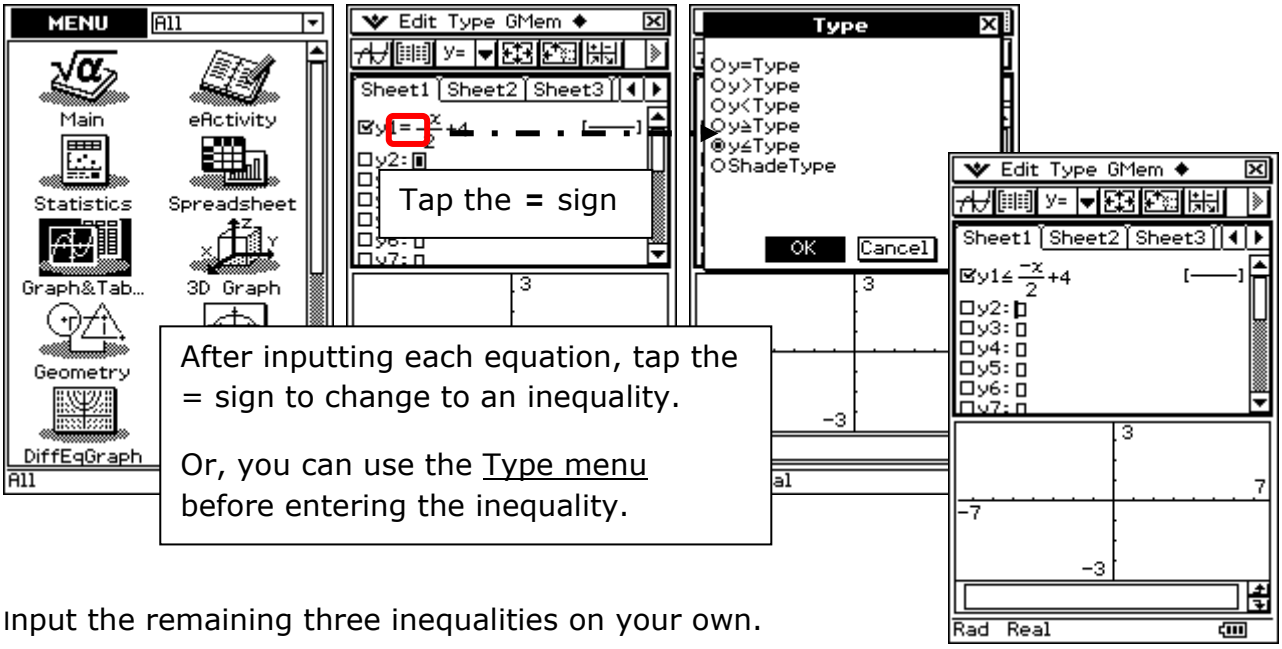
Find the maximum and minimum value for z = 2x + 5y if it is subject to the following constraints:

$$\left\{\begin{array}{l} x+2y\le 8 \\ 4x-2y\ge 6 \\ x-2y\le 4 \\ y\ge 0 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} y\le \dfrac{-x}{2}+4 \\ y\le 2x-3 \\ y\ge \dfrac{x}{2}-2 \\ y\ge 0 \end{array}\right\}$$

To graph these on the ClassPad, we need to first solve each inequality for y.

The four inequalities listed above are the <u>constraints</u> for z=2x+5y. The shaded area that they mark off when graphed will be the <u>feasibility region</u>. We need to find the <u>corner points</u>, (x, y), of the region that yield the largest and smallest values of z.

Next, open the Graph Editor and input each inequality:



After inputting each equation, tap the = sign to change to an inequality.

Or, you can use the <u>Type menu</u> before entering the inequality.

Input the remaining three inequalities on your own.

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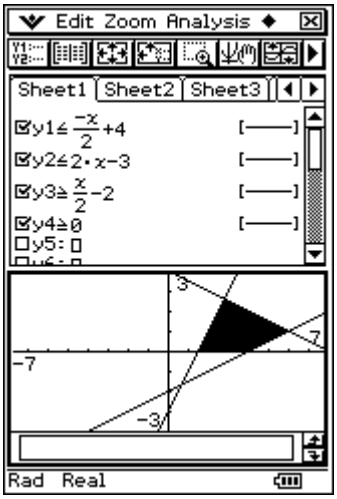
Tap the $ button to see the feasibility region (shaded part). You should see:

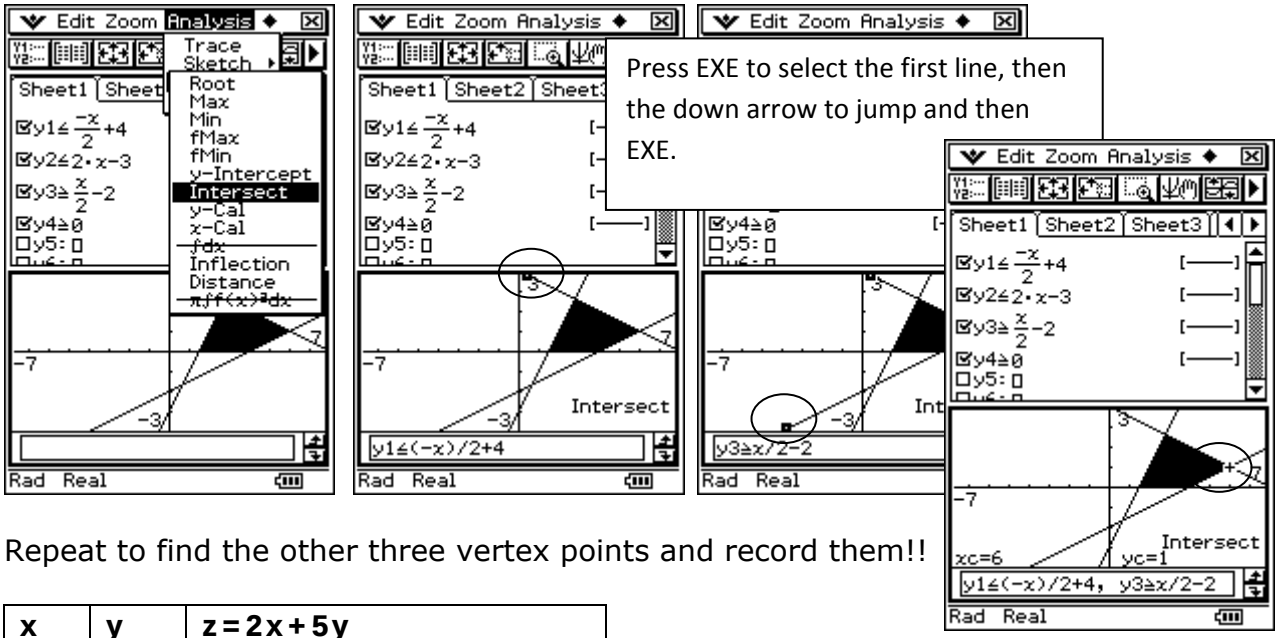


Optimal conditions for a maximum or minimum will occur at one of the vertex points.

Next, we need to find and record each vertex point to find the optimal solutions.

a) Tap Analysis/G-Solve/Intersect
b) Press EXE to select a line OR the down arrow on the cursor pad to jump to another line.

   Hint: The blinking box lets you know where you are, pressing EXE selects it. Two lines need to be selected to get an intersection.



Press EXE to select the first line, then the down arrow to jump and then EXE.

Repeat to find the other three vertex points and record them!!

x	y	z=2x+5y
6	1	Z=2(6)+5(1)=17
2.8	2.6	Z=2(2.8)+5(2.6)=18.6
1.5	0	Z=2(1.5)+5(0)=3
4	0	Z=2(4)+5(0)=8

So, we have the **maximum** of z is 18.6 and occurs at (2.8, 2.6).

The **minimum** of z is 3 and occurs at (1.5, 0).

Exercise 1

A farm with limited workers has 15 acres to plant. They have 100 hours to plant and 125 hours to harvest the crops. Their plan is to grow cucumbers and tomatoes. It takes 6 hours to plant and 4 hours to harvest an acre of cucumbers. It takes 10 hours to plant and 15 hours to harvest an acre of tomatoes. Each acre of cucumbers produces $700 and each acre of tomatoes produces $900 in revenue.

How many acres of each should be planted to maximum profit?

Let x=number of acres of cucumbers

Let y=number of acres of tomatoes

Organize information in a table:

	x	y	Total	Constraints	Solve for y
Acres of Land	x	y	15	$x+y\leq 15$	$y\leq -x+15$
Time to Plant	6x	10y	100	$6x+10y\leq 100$	
Time to Harvest	4x	15y	125	$4x+15y\leq 125$	
Natural constraint				$y\geq 0$	
Natural constraint				$x\geq 0$	

a) Solve each constraint for y and record in the table above.

b) Graph the inequalities and record the coordinates of each vertex point in the table below. For the vertex on the y-axis, use Analysis/G-Solve/y-Intersect.

c) How much profit does each optimal point produce?

(x,y)	Profit=700x+900y	Maximum?
(0,0)	Profit = 0	No (nothing planted)

d) Explain how many acres of each should be planted to maximum profit.

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Exercise 2
A gold processor has two sources of gold ore, source A and source B. In order to keep his plant running, at least 3 tons of ore must be processed each day. Ore from source A costs $20 per ton to process, and ore from source B costs $10 per ton to process. Costs must be kept to less than $80 per day. Moreover, Federal Regulations require that the amount of ore from source B cannot exceed twice the amount of ore from source A. If ore from source A yields 2 oz. of gold per ton, and ore from source B yields 3 oz. of gold per ton, how many tons of ore from both sources must be processed each day to maximize the amount of gold extracted subject to the above constraints?

In this case they are looking for the number of tons of ore from the two sources which should be processed each day to maximize the amount of gold extracted.

Step 1. Define the unknowns

Let x = the number of tons from source A

Let y = the number of tons from source B

Step 2. Express the objective and the constraints.

The objective is to maximize the amount of the gold yield. Since each ton of ore from source A yields 2oz. of gold and each ton of ore from source B yields 3oz. of gold, the amount of gold recovered will be...

Amount of Gold =

After getting the unknowns and the objective out of the way, everything else in the problem is a constraint.

Organize information in a table and solve each constraint for y.
Record in the table below.
(Fill in the relevant missing pieces):

	x	y	Total	Constraints	Solve for y
Processing	x	y	3	x+yù3	y≤-x+3
Cost	20x	10y	80		
Federal Regulations	2x	y		y≤2x	
		Natural constraint		y≥0	
		Natural constraint		x≥0	

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Step 3. Graph the constraints.

Graph the inequalities and record the coordinates of each vertex point in the table below. For the vertex on the y-axis, use Analysis/G-Solve/y-Intersect.

Step 4. Evaluate all optimal points.

a) How much gold does each optimal point produce?

(x,y)	Amount of Gold=2x+3y	Is it a maximum?

b) Explain how many tons of ore should be processed from each source to maximize the amount of ore extracted.

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Exercise 3
A builder has 42 units of material and 32 units of labor available for use during a given period and can build houses of type A or of type B or some of both. How many of each should he build so as to make a maximum profit if he makes $1,000 on each house of type A and $400 on each house of type B? Assume that a house of type A requires 7 units of material and 1 of labor, whereas a house of type B requires 1 unit of material and 2 units of labor. What is the maximum profit?

Step 1. Define the unknowns

Let x =

Let y =

Step 2. Express the objective and the constraints.

The objective is to maximize the profit. Express the objective with an equation:

Profit =

After getting the unknowns and the objective out of the way, everything else in the problem is a constraint.

Organize information in a table and solve each constraint for y.
Record in the table below:

	x	y	Total	Constraints	Solve for y
Natural constraint				y≥0	
Natural constraint				x≥0	

Step 3. Graph the constraints.

Graph the inequalities and record the coordinates of each vertex point in the table below. For the vertex on the y-axis, use Analysis/G-Solve/y-Intersect.

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Step 4. Evaluate all optimal points.

c) How much profit does each optimal point produce?

(x,y)	Amount of Gold=2x+3y	Is it a maximum?

d) Explain how many of each type of house should be built to maximize his profit.

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Exercise 4

Use the Classpad to find the maximum and minimum values of the objective function, and the coordinates of the vertices at which they are attained.

1. $x \geq 0, y \geq 0, 3x + 7y - 21 \leq 0, f(x) = 2x + 3y + 1$

2. $x \geq 1, y \leq 2, 2x + y - 2 \geq 0, f(x) = 3x - 5y + 6$

3. $x \geq 0, x + 2y - 4 \leq 0, x - 2y \leq 0, f(x) = -x + 4y - 3$

4. $x \leq 0, y \geq 0, 2x + y + 2 \geq 0, x - 2y + 6 \geq 0, f(x) = 5x - y - 1$

5. $x + 3y - 3 \leq 0, 3x - y - 9 \leq 0, x + 4y + 10 \geq 0, 3x - 2y + 2 \geq 0, f(x) = 2x - 7y - 5$

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