**Name:**_____ **Date:**_____

# Applications of the Derivative

**Introduction to differentiating:**

Motion is a continual change in position. Suppose you set out to drive to your cousins wedding. You leave at 9am having to quickly return home because you forgot their gift. Then on the way there you drive through Portland's peak traffic. Unfortunately as soon as you get there you remember that you left the water running and all the lights on since you left the house in a frenzy, so you drive back home right away. You arrive at your house at 3:45pm. Your odometer now has 82 miles more than when you started driving. What was your average speed during these hectic 6.75 hours?

_____MPH

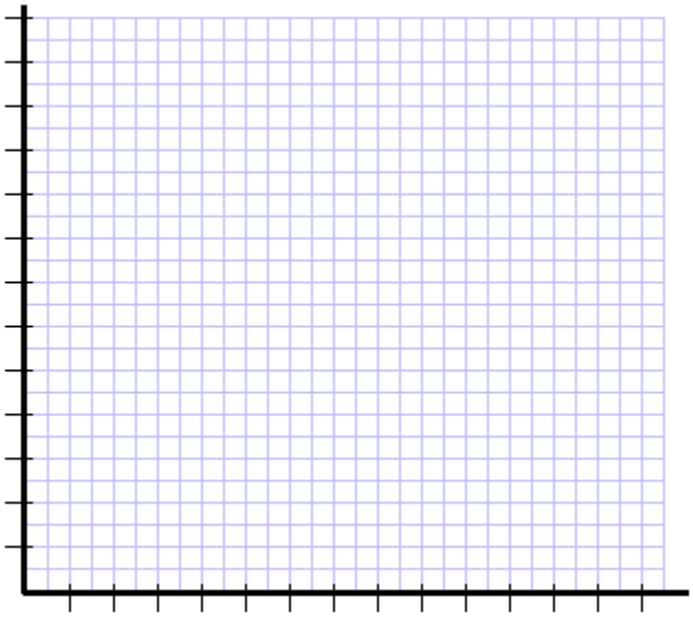
Realistically you were not driving this speed all the time. On the way home you were probably going 30MPH.

1. Suppose the function that describes your position relative to time for this trip is:

$f(x) = -.0045x^5 + .12x^4 - 1.1x^3 + 4.28x^2 - 4.32x + 1.94$

Sketch the graph of your drive for $0 < x < 14$

x = Time (2 marks =1hour)
y = Distance (1 mark=4miles)



To Graph: a. From the start menu (m) select (W).
b. Type the formula with the soft keyboard (k) then press E.
c. Tap $ to graph the function.
d. Select **O/View window** and set xmin = 0, xmax = 14, ymin = 0 and ymax = 42 Tap OK.

To find intercepts exactly:

i. Tap on the graph to give it focus.
ii. To get the y-intercept, go to **Analysis/G-Solve/y-Intercept**.
iii. To get the x-intercept, go to **Analysis/G-Solve/Root.**
iv. To get the maximum point, go to **Analysis/G-Solve/Max.**
v. You can trace the graph by going to **Analysis/Trace.**

2. Find your average speed during the last 30 minutes of your drive i.e. on the interval x=12 to x=13
   a. To find the y coordinate at each x coordinate tap the graph to give it focus, then select **Analysis/G-Solve/y-Cal**
   b. Enter an x value. **(x-Cal** gives you the x values when you give the y values**.)**

   The slope through points ( 12, ) and ( 13, ) is $m =$

3. How fast were you going exactly 2 hours after you began the trip?

To find the average speed we had to look at an interval of time. To find how fast you were going at a certain instant in time you have to make the interval of time infinitesimally small. This means: step off of the 2 hour marker an arbitrary distance $\Delta x$ and find the limit of the slope of the secant lines as $\Delta x \to 0$. Hence we get an expression known as the **derivative** of a function $f(x)$ at any value of $x$. Hence the question becomes: What is the derivative of $f(x)$ at x = 4 (which is 2 hours on the grid). The below equation gives equivalent forms of the derivative.

$$\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{d}{dx}(f(x)) = f'(x)$$

4. Evaluate $\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$ for $f(x) = -.0045x^5 + .12x^4 - 1.1x^3 + 4.28x^2 - 4.32x + 1.94$ at $x = 4$ (2 hours)

Can you imagine doing this by hand?!!!

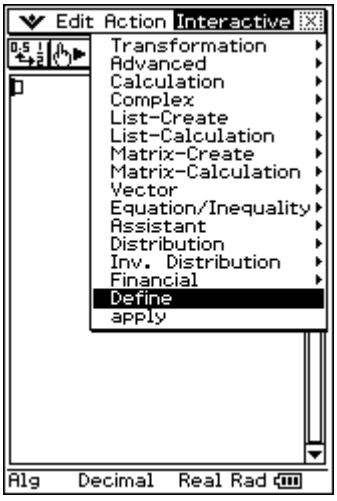
a. Select the expression within the Graph editor menu. Select **Edit/Copy.**

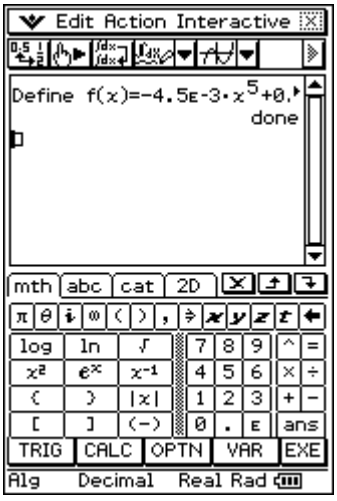
b. From the start menu (m) select (J). Select **Edit/Clear all** if needed.

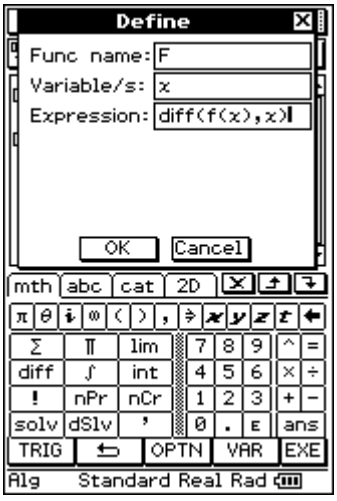
c. Under **Interactive/Define** define your function by entering in the expression**.**
Enter the formula by tapping H on the keyboard (k) then tap **OK.**

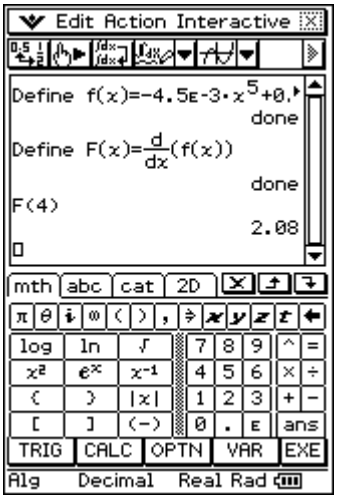
d. Define the derivative by selecting **Interactive/Define** again**.** Now type in **F** for the derivative function name and type in diff(f(x),x) for the expression. (You can also select **"diff"** from the 9 tab under -.) Tap OK.

e. Finally find the derivative of $f(x)$ at $x = 4$ by evaluating F(4) (You may **select, drag & drop** with the stylus from any line)




Find how fast you were going 2.5 hours after you started.

**F(5) =**

At what time(s) does the graph show you stopped? Find these values of $t$. i.e find **F(x) = 0.**

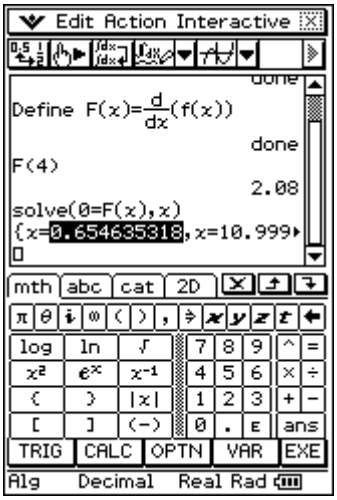
i. To solve an equation in J select **Interactive\Advanced\Solve.**

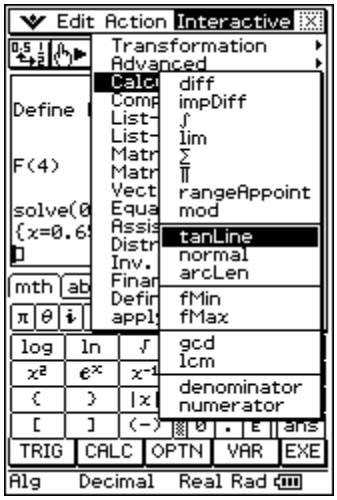
ii. Under equation type input 0 = F(x) (we use x variable for the x axis). Tap OK

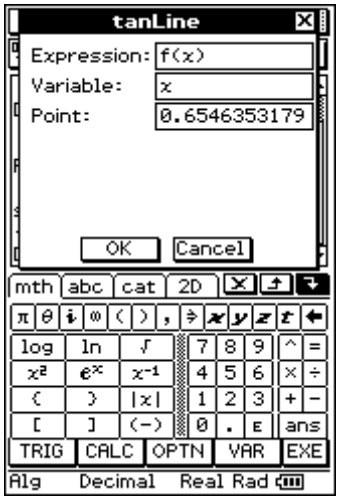
$t_1 =$ $t_2 =$

What is the slope of the tangent line at these places?

i. To find the tangent line at any point in J select the number with your stylus and press the copy G button.
ii. Next, tap on the new line and then select **Interactive/Calculation/tanLine.**
iii. Under expression type in f(x). Tap in the point field and press H to paste the number you copied. Tap OK.




Find the slope of the tangent line at the other x value.
Why is the slope of the tangent line equal to zero at these times? (Note: if you get a number like: -2.732E-12 that is considered to be zero due to round off error.)

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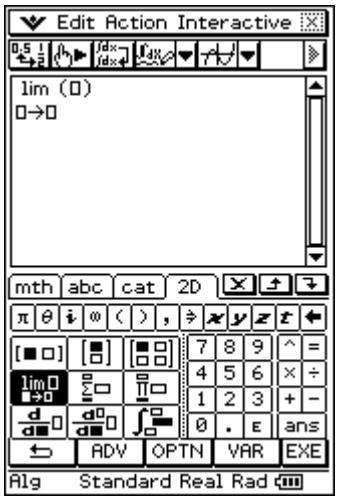
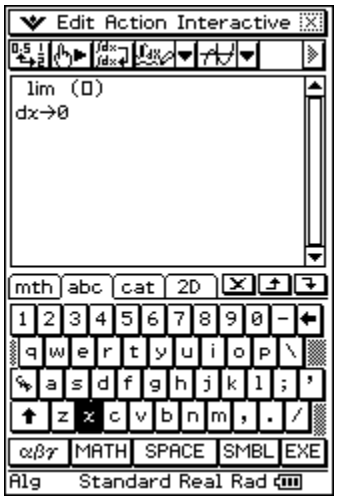
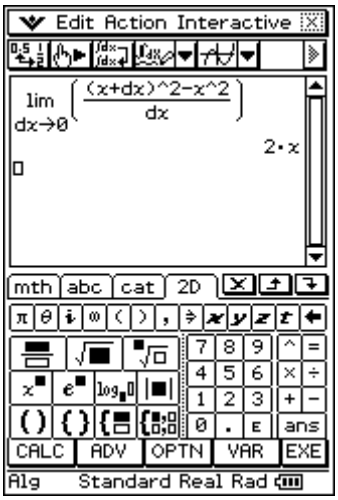
**Optional:**
To find the derivative of your function you may also use the limit notation ; in the soft keyboard under ) tab in the - option menu. Note: instead of entering a $\Delta x$ from the 0 menu enter **dx** ("deltax") into the limit. For example if your function was $f(x) = x^2$,

i. In J select **Edit/Clear All**. Tap ) then -option to find ;.
ii. In the Soft Keyboard k select 0 and **tap** letters "dx" without spaces.
iii. Type in the remaining formula and tap E.




**Differentiation:**
1. Your castle is under siege, but luckily you have the advantage of tall walls from which to throw burning oil

at the soldiers below. The height of the oil with respect to time can be modeled by the following function: $f(t) = -16t^2 + 32t + 128$ where $t$ is in seconds. If a beaten soldier laying on the ground below opens his eyes to see a barrel of burning oil being overturned at the top of the wall, how many seconds does he have to get up and run away before the oil hits him? (Careful with the negative sign)

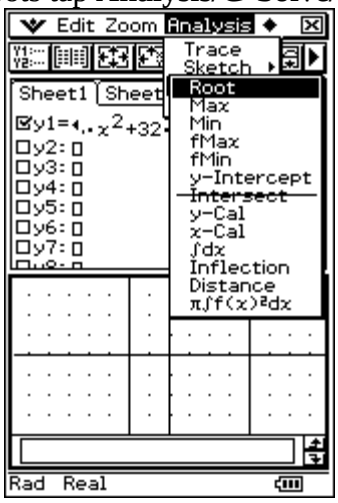
Find the positive root of the parabola by graphing $f(t)$.

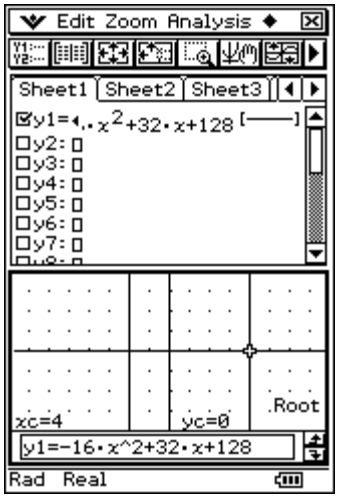
To Graph: a. From the start menu (m) select (W). Select **Edit/Clear All** if needed.

b. Type the formula with the soft keyboard (k)
then press E.

c. Tap $ to graph the function.

d. To find the roots tap **Analysis/G-Solve/Root.** Use the cursor arrows to see all zeros (roots).




2. Suppose the soldier is too badly beaten and doesn't get up in time, with what speed is the burning oil going to hit him as he lays on the ground? Find the derivative of the position function to get a function giving you the speed of the oil at any time $t$.

a. From the start menu (m) select (J). Select **Edit/Clear all** if needed.

b. Under **Interactive/Define** define your function by entering it in to the expression field**.** Enter in the formula with the soft keyboard (k) then tap **OK**.

c. Define the derivative by selecting **Interactive/Define.** Now type in **F** for the derivative function and type in diff(f(x),x). (You can also select it under the 9 tab under -) Then tap **OK**.

d. Evaluate the derivative of $f(x)$ by evaluating F(x) at x = 4. Type F(4) and press E.