Name:___________________________________ Date:__________________

Implicit Differentiation

Recall that the definition and the notations of the derivative of a function are given by:

$$y' = \frac{dy}{dx} = \frac{d}{dx} f(x) = f'(x) = \lim \frac{f(x+h) - f(x)}{h} \text{ as } h \to 0$$

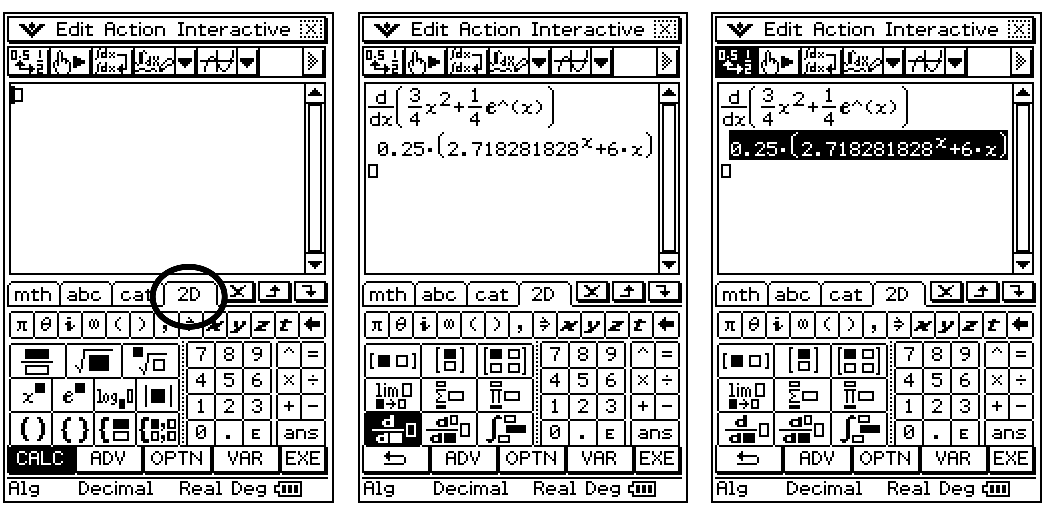
Exercise:
Find the derivatives of the following equations.
Hint: First solve for y if possible, then input that expression as directed below.

1. $y = 3x^2 + e^x - 3y$. $\qquad y' =$

2. $5\sqrt{y} - \ln(x) = (2x+1)^2 + \sqrt{y}$ $\qquad y' =$

3. $\dfrac{-\cos^2(x)}{y+1} = y - 1$ $\qquad y' =$

1. From the start menu (m) select J. Select **Edit/Clear All** if needed.
2. In the soft keyboard (k) under) under the □ page select] and then type your function in. Tap E. (Remember to use e from the 9tab. Tap I to return back to previous page within a tab.)
3. Select the output and then tap u to rewrite as fractions and exact numbers. (Remember that differentiation works with real numbers. This is why you must be in radian mode or else a $\frac{\pi}{180}$ will be included the result.)



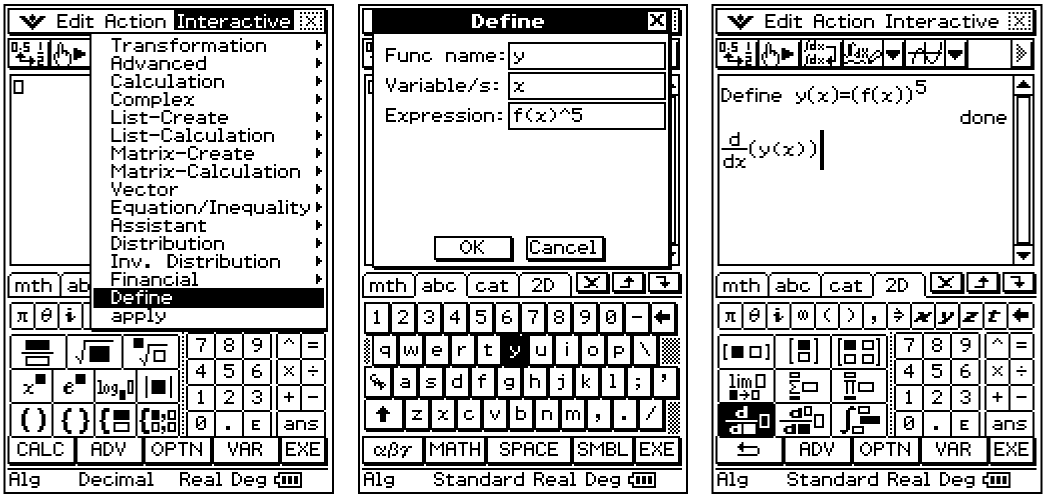
In the previous cases it was easy to find the derivative because each was solvable for the variable y. What if we didn't know the formula or could not solve the equations for the usual variable y? See next example.

Find the derivatives of the follow equations and expressions.

4. $y = [f(x)]^5$ $\qquad\qquad$ $y' = \dfrac{d}{dx}[f(x)]^5 =$

1. From the start menu (m) select J. Select **Edit/Clear All** if needed.
2. Select **Interactive/Define.** Input y for **Func name** and $f(x)$ for **Expression** from the 0tab. Tap **OK.**
3. In the soft keyboard k under) select] and then type your function in. Tap E.

(Remember to tap I to return back to previous page within a tab.)



Why are there two $f(x)$?

Without plugging it into your calculator determine the derivative of this function:

5. $y = [y(x)]^5$ (Chain rule) $\qquad\qquad$ $y' = \dfrac{d}{dx}[y(x)]^5 =$

When an equation isn't solved for a variable (i.e. $y =$) then it is called an *implicit equation.*
When it is solved for a variable it is called an *explicit equation.*
Circle the equation(s) that are written explicitly.

a. $\dfrac{-\cos^2(x)}{y+1} = y$ $\qquad$ b. $y = \pm\sqrt{x^2+1}$ $\qquad$ c. $y - 2 = x^3$ $\qquad$ d. $16 - (x-3)^2 = y^2$

It is easiest to find the derivative of a function when it is in explicit form (solved for y).
How would you know if an equation can be written explicitly?
What must be true about the exponent on every term that includes the variable y?

When an equation can't be written explicitly then we must use implicit differentiation.

This requires that we:

i. Take the derivative with respect to x of every term in the equation (both sides of equals sign).

ii. When differentiating a term that has a y variable remember to use the chain rule and differentiate y as a function of x :(i.e. $y(x)$).

iii. Collect all y' or $\frac{dy}{dx}$ and isolate it on one side of the equation.

Why does the chain rule need to be applied to each term with variable y? Is the variable itself a composition of functions?

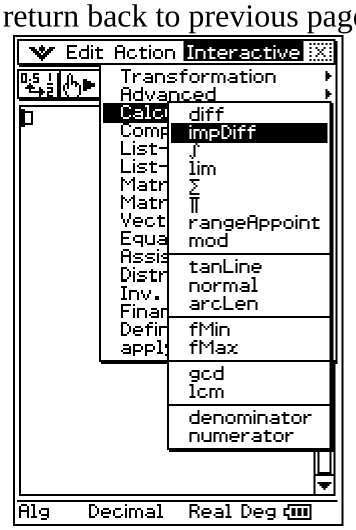
Exercise: Differentiate the following formulas by hand, then use the ClassPad to check your work.

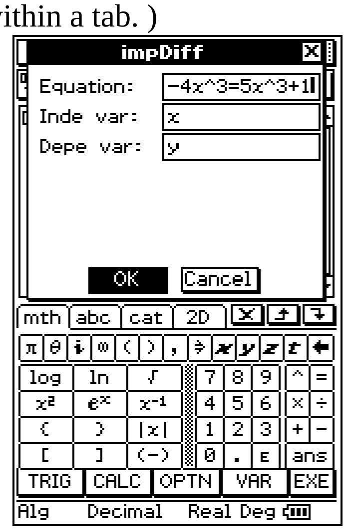
6. $y^4 - 3y - 4x^3 = 5x^3 + 1$ $\qquad y' = {}^{dy}\!/_{dx} =$

7. $xy = \sin(y) + x^2 y^2$ $\qquad y' = {}^{dy}\!/_{dx} =$

1. From the start menu (m) select J. Select **Edit/Clear All** if needed.
2. Select **Interactive/Calculation/impDiff.** Input equation using soft keyboard k. Tap **OK**.

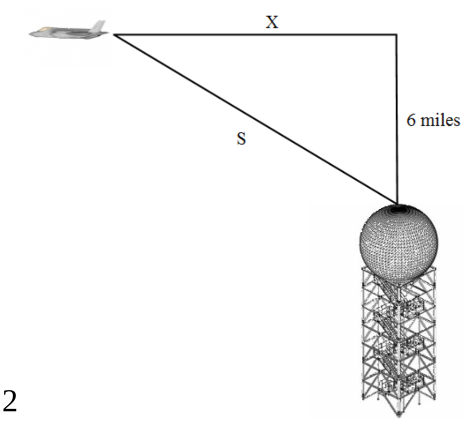
(Remember to tap I to return back to previous page within a tab.)





Applications of implicit differentiation are usually found in problems where two rates of change are related by a formula, i.e. related rates.

1. A state-of-the-art fighter jet plane flying at an altitude of 6 miles passes directly over a radar antenna. When the plane is 12 miles away (slant distance: $s=12$ miles), the radar detects that the distance s is changing at 750 mph. What is the speed of the plane?



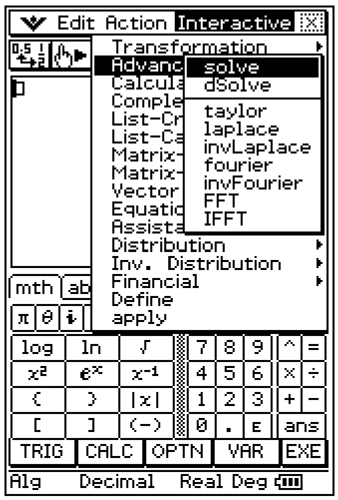
i. What is changing? For which do we have numerical values?

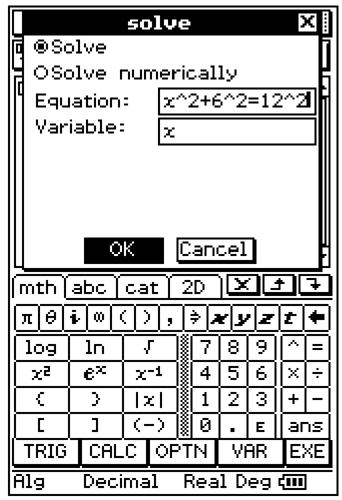
$\frac{dx}{dt}=$ and/or $\frac{ds}{dt}=$

ii. What right triangle theorem relates x and s? When $s=12$ what is x?

$x=$

1. From the start menu (m) select J. Select **Edit/Clear All** if needed.
2. Select **Interactive/Advanced/Solve.** Input the equation replacing $s=12$
3. Input x for **Variable.** Tap **OK.**





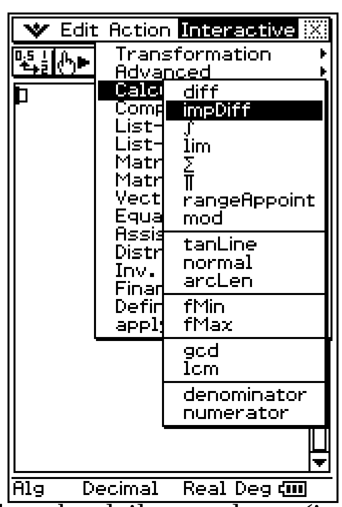
iii. Differentiate the equation implicitly with x being the independent variable and s (from the 0tab) being the dependent variable.

1. Select **Interactive/Calculation/impDiff.** Input equation using soft keyboard k. Tap **OK.**
2. Determine the numerical value of s' with respect to x by inputting values for x and s that you know.

$\frac{ds}{dx}=$

iv. Fill in the expression to determine $\frac{dx}{dt}$

$$\frac{dx}{dt}=\frac{\frac{ds}{dt}}{\frac{ds}{dx}}=\quad-$$



v. The plane is traveling at ___________ **mph.**

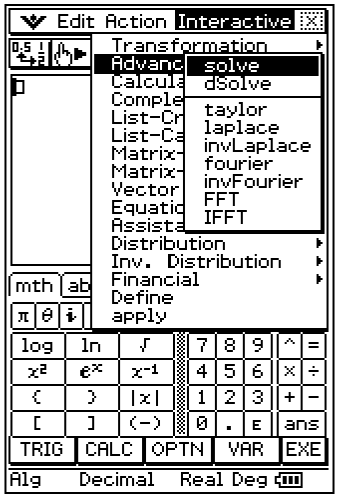
2. The price of gasoline g (in dollars per gallon) is related to the daily supply x *(in thousands of gallons)* by the equation $500g^2-x^2-100=0$. If 20,000 gallons of gasoline are available for a certain day, and the price is falling at the rate of 2 cents per gallon per day, at what rate is supply falling?

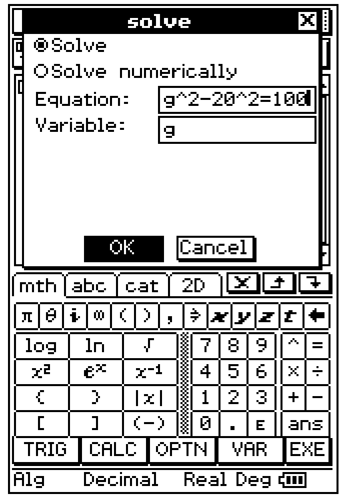
i. What two variables are changing? For which do we have numerical values?

$\frac{dg}{dt} =$ and/or $\frac{dx}{dt} =$

ii. What equation relates g and x? When $x = 20$ what is g? $g =$

1. From the start menu (m) select J. Select **Edit/Clear All** if needed.
2. Select **Interactive/Advanced/Solve.** Input the equation replacing $x = 20$.
3. Input g for **Variable.** Tap **OK.**

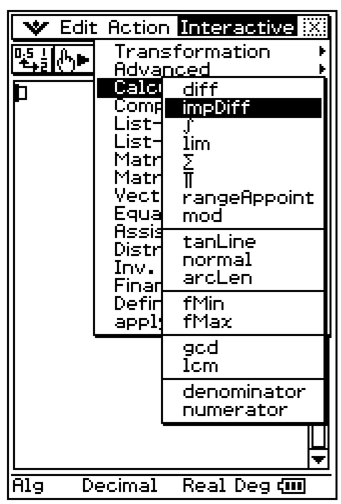




iii. Differentiate it implicitly with x being the **independent** variable and g (from the 0tab) being the **dependent** variable.

1. Select **Interactive/Calculation/impDiff.** Input equation using soft keyboard k. Tap **OK**.
2. Determine the numerical value of g' by inputting values for x and g that you know.

$\frac{dg}{dx} =$



iv. Fill in the expression to determine $\frac{dx}{dt}$

$$\frac{dx}{dt} = \frac{\frac{dg}{dt}}{\frac{dg}{dx}} = -$$

v. The supply is falling at a rate of __________ **gallons per day.**