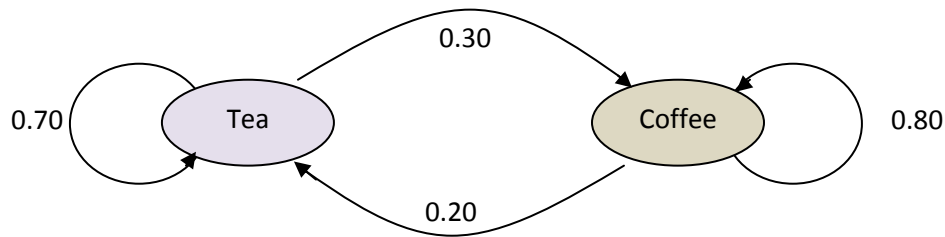


Markov Chains

Example

Name _____

An office building gives out free hot drinks to its employees. The options are tea and coffee. Each day 70% of the people who drank tea yesterday choose to drink tea again today and 30% of the people who drank tea yesterday switch to coffee. Also, 80% of those who drank coffee yesterday choose coffee again today while 20% of those who drank coffee yesterday switch to tea.



If one day there are 120 people drinking tea and 80 people drinking coffee, how many people will be drinking tea 1 day later? 2 days later?

Solution:

The number of people drinking tea the next day will be 70% of those drinking tea today plus 20% of those drinking coffee today.

$$0.70(120) + 0.20(80) = 100$$

Similarly we can find the number of people drinking coffee tomorrow.

$$0.30(120) + 0.80(80) = 100$$

We can summarize these two equations with a single matrix equation.

$$\begin{bmatrix} 0.70 & 0.20 \\ 0.30 & 0.80 \end{bmatrix} \begin{bmatrix} 120 \\ 80 \end{bmatrix} = \begin{bmatrix} 0.70(120) + 0.20(80) \\ 0.30(120) + 0.80(80) \end{bmatrix} = \begin{bmatrix} 100 \\ 100 \end{bmatrix}$$

Let's label what we have thus far.

$$P = \begin{bmatrix} 0.70 & 0.20 \\ 0.30 & 0.80 \end{bmatrix}, \quad \mathbf{x}_0 = \begin{bmatrix} 120 \\ 80 \end{bmatrix}, \quad \mathbf{x}_1 = \begin{bmatrix} 100 \\ 100 \end{bmatrix}$$

We can see that each day that goes on we apply the matrix P to yesterday's state. We have $x_1 = Px_0$ and in general:

$$x_{k+1} = Px_k \text{ for } k=0,1,2,\dots$$

To see the distribution on the second day we apply P to x_1 .

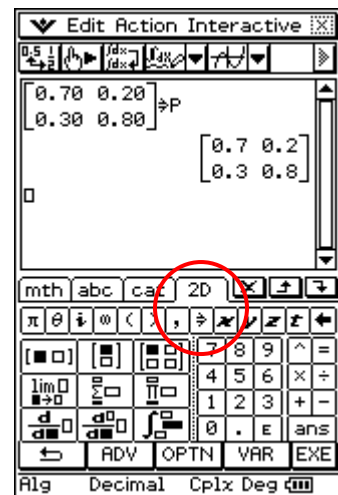
$$x_2 = Px_1 = \begin{bmatrix} 0.70 & 0.20 \\ 0.30 & 0.80 \end{bmatrix} \begin{bmatrix} 100 \\ 100 \end{bmatrix} = \begin{bmatrix} 90 \\ 110 \end{bmatrix}$$

Exercise 1:

What will the distribution be in 5 days? 20?

Notice that since $x_2 = Px_1$ and $x_1 = Px_0$, then $x_2 = P(Px_0) = (PP)x_0 = P^2x_0$.

1. Open M on your ClassPad.
2. Open your Keyboard, go to the 2D tab, then CALC, tap the 8 to create the matrix $\begin{bmatrix} 0.70 & 0.20 \\ 0.30 & 0.80 \end{bmatrix}$.
3. Find the Store button, $\boxed{\rightarrow}$.
4. Open the "abc" tab and select P.
5. Hit EXE.
6. Raise P to the desired power and multiply it to x_0 .



Exercise 2:

How many people will be drinking tea on day two if day zero had 80 of the 200 people drinking tea?

What about day 20? (Hint: That means $\mathbf{x}_0 = \begin{bmatrix} 80 \\ 120 \end{bmatrix}$.)

Exercise 3:

A school of 300 people has a hot lunch option in the cafeteria. If students do not use that option, they instead bring a sack lunch. Each day 15% of the people who had hot lunch yesterday switch to having sack lunch. Also, 10% of the people who yesterday brought a sack lunch switch to eating hot lunch. If everyone eats lunch every day and yesterday 200 people ate hot lunch, how many people will be eating hot lunch in 5 days?