

Name: _____

Date: _____

Law of Sines

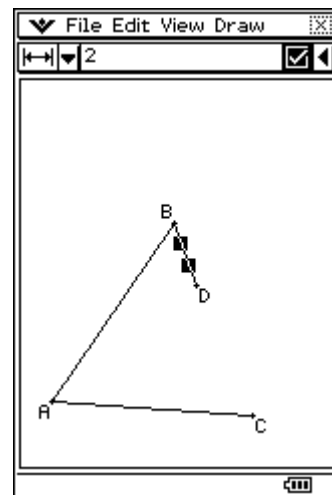
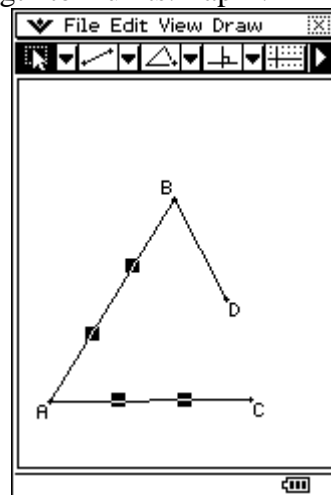
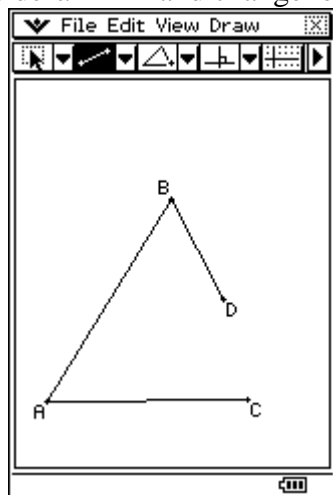
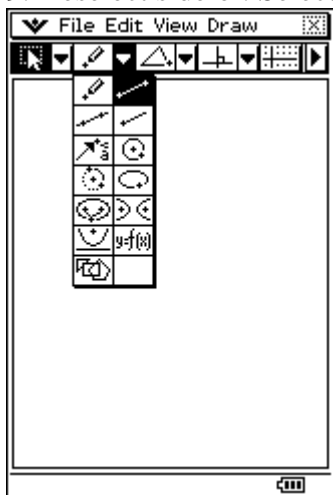
Recall that the law of sines is given by

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c} \quad \text{or} \quad \frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$


Suppose you go for a morning jog and a police officer stops you because you are running too fast! How would you feel? What would you say to the police officer? "The speed limit law doesn't apply to joggers." In a similar way, despite being called the *law* of sines, this law can't always be applied.

Construct the triangle with $m < A = 60^\circ$, $a = 2$, and $b = 6$.

1. From the start menu (m) select (G). Tap **Edit/Clear All**.
2. Tap the 2nd n and select y.
3. Tap once in the lower left hand corner and in the upper middle of the screen.
4. Tap the point A (making sure the status bar says "Snap to A") and tap to the right of point A.
5. Tap point B and tap again below point B.
6. Select G. Tap sides AB and AC.
7. Tap u (far right) and change angle measure to 60° . Tap E execute. Make sure s highlighted.
8. Tap in clear space to deselect sides. Tap on side $b = AC$ and change its length to 6 units. Tap E.
9. Deselect side b . Select side $a = BD$ and change length to 2 units. Tap E.



With the given measurements in place, is there any way that point D and C will coincide? Select only point D. Now tap and drag point D to point C without lifting the stylus. What happened?

(Use the cursor arrows  to put the triangle back into the center of your screen.)

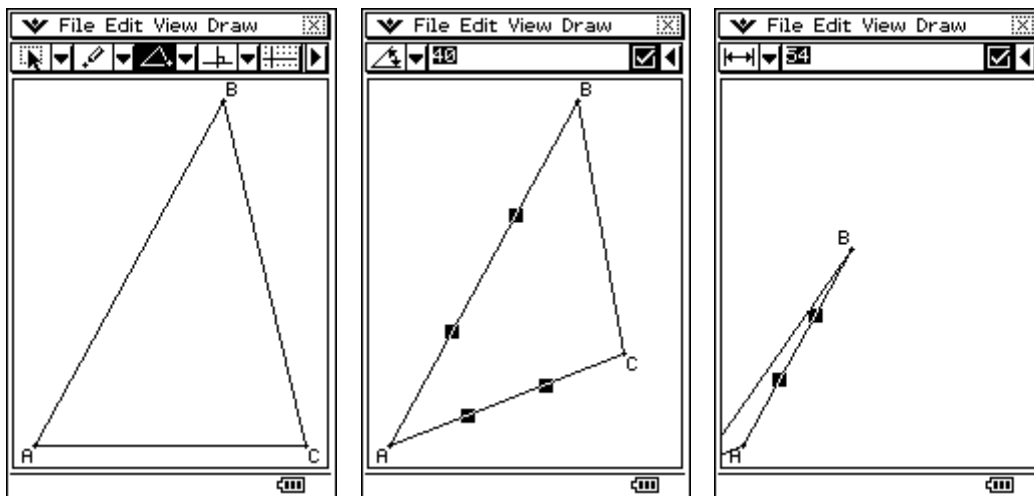
We cannot apply the law of sines here because a triangle can't exist having the above measurements.

On the other hand if you were asked for which angle θ does $\sin(\theta) = \frac{1}{2}$, you would say that there are two

possibilities since it depends on the quadrant that you are in. For this reason the law of sines could give two different measurements because there are two possible triangles. One would have $\theta < 90^\circ$ and the other $\theta > 90^\circ$.

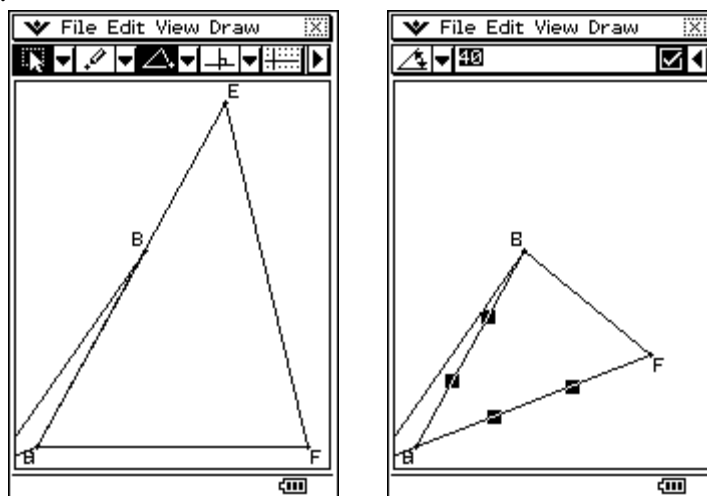
Construct the triangle with $m < A = 40^\circ$, $a = 62$, and $c = 54$.
(We will define the angle and lengths in this order.)

1. From the start menu (m) select (G). Tap **Edit/Clear All**.
2. Tap O on the top menu bar and then tap once anywhere in the geometry window.
3. Tap on side AB and then tap on side AC. With the selection handles showing tap u.
4. Tap on 61.21479 and change it to 40 and press E.
5. Tap in clear space to deselect the angle. Tap on side BC, tap on the menu bar and enter 62; tap E.
6. Deselect side BC and select side AB. Tap on the menu bar and enter 54; tap E.



Without moving the triangle you just made, construct the same triangle by inputting the same data in this order:
 $a = 62$, $c = 54$, $m < A = 40^\circ$

1. Tap w and then tap O and then tap anywhere in the geometry window. (Note that the ClassPad names the angles DEF instead of ABC, but we will use the same orientation.)
2. Select segment DE by tapping near point E, then tap u and change the number to 54.
3. With the selection handles still on DE tap segment DF and make the angle 40.
4. Deselect everything and select side EF and make it 62. Deselect everything
5. Press - once to zoom out.



- a. What are the two possible angles for A? How many triangles do you see that include A as a vertex point?
To see the angle tap on side AC and then on AB. (You might need to zoom in again)
Deselect everything and tap on segment DE and then on DF.

b. Now use the Law of Sines to come up with the two possibilities for angle B.
At what point in solving for angle B do you come up with 2 possibilities? Why?

c. Determine the equality/inequality sign ($=, >, <$) that fits in each case. Write them above each question mark.

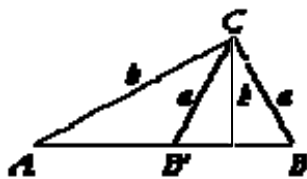


Fig. 1

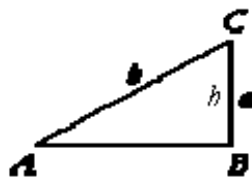


Fig. 2

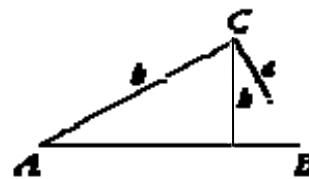


Fig. 3

$\frac{h}{a} ? 1$, which implies $h ? a$ (Fig. 1)

$\frac{h}{a} ? 1$, which implies $h ? a$ (Fig. 2)

$\frac{h}{a} ? 1$, which implies $h ? a$ (Fig. 3)

d. Given information about the triangle (i.e. angle A and side b from the figures) we have a right triangle that can be made with sides h, b, c. What trigonometric function should be used to find h in terms of angle A and side b?

Solve for $h = ?$

Exercises:

a.) Without using the law of sines determine whether there are 0, 1, or 2 possible triangles.
Then use the law of sines to find $m < C = ?$

$\triangle ABC$ has measurements: $m\angle A = 151^\circ$, $a = 412$, and $c = 342$.

b.) Find all of the possible measurements of the triangle by first determining how many triangles are possible given the following measurements.

$\triangle ABC$ has measurements: $m\angle A = 43^\circ$, $a = 31$, and $b = 37$.

c.) APPLICATIONS: A helicopter awkwardly finds itself between two enemy battle ships that are 1600 ft apart. The angle of elevation from the first ship is 40° , and from the second ship is 28° . At this moment if it is shot down by a missile how far down will it fall to the ocean? (What is the helicopter's height above sea level)
Hint: Draw a line through the helicopter that is parallel to sea level. Use alternate interior angles.

